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| Fixed income securities  Lecture notes  O.D. Lecturing Legacy |

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**1 ESSENTIALS OF BOND PRICING**

**1.1 Straight bond**

straight bond (conventional bond, plain vanilla bond, bullet bond) pays its holder a fixed coupon with a given frequency (usually semi-annually) over a given period of time (longer than one year) and the principal (nominal value, face value, par value) at the maturity of the bond

**1.2 Classification of bonds**

bonds form a group of securities with a wide range of properties that can be classified according to a large number of criteria

**i) Length of coupon period**

* a straight bond pays coupons with a given frequency (at the end of every six months, every quarter, every year etc.)
* zero-coupon bonds do not pay coupons at all, the reward for holding the bond has the form of a discount from the nominal value when the bond is purchased

**ii) Size of coupon payments**

* a straight bond always pays the same coupon, which is determined as a percentage (called coupon rate) of its principal and is not adjusted to the exact number of days in the coupon period
* variable-coupon bonds link the coupon to some economic variable such as the retail price index (inflation-linked bond), money market interest rate (floating rate note) and others
* collateralised bonds derive coupons from a cash flow of a given package of underlying assets (asset-backed securities)
* income bonds make coupon payment only if the income generated by the issuing firm is sufficient

**iii) Redemption date**

* a straight bond has one redemption date at maturity (bonds are referred to according to their times to maturity: five-year bond, ten-year bond and so on)

US terminology: Treasury bill (maturity does not exceed one year), Treasury note (maturity is between 1 and 5 years), Treasury bond (maturity is over 5 years)

* callable bonds give their issuers the right to retire the bond before its stated maturity under specified conditions (call price, beginning of callable period)
* puttable bonds give their holders the right to choose the actual date of redemption
* perpetual bond (perpetuity, consol) has no official redemption date so the coupon is paid perpetually
* convertible bonds give their holders the right to convert the bond at maturity into other bonds or equity

**iv) Issuer of the bond**

* government bonds are issued by sovereign governments with the aim to finance public deficit and manage public debt (called Treasuries in USA, gilts in UK)
* municipal bonds are issued by local authorities
* corporate bonds are issued by private companies (senior vs. junior debt, secured vs. unsecured debt)

**v) Currency denomination**

* domestic bonds are issued by resident bodies in a resident currency (German corporation issues a bond on the German capital market denominated in the euro)
* foreign bonds are issued by non-resident bodies in a resident currency (British corporation issues a bond on the German capital market denominated in the euro)

nicknames of foreign bonds according to national capital markets: Samurai (Japan), Yankee (USA), Bulldog (UK), Matador (Spain), Kiwi (New Zealand), Alpine (Switzerland), Panda (China)

* eurobonds are issued in a currency which is different from the resident currency (German corporation issues a bond on the German capital market denominated in US dollars); the terminology has nothing to do with the euro as a currency

**vi) Credit risk**

credit risk is reflected in the rating granted by rating agencies (Standard & Poor’s, Moody’s, Fitch)

the lower the credit rating, the greater the risk of default and the higher the risk premium

classification of grades: investment grade, non-investment (speculative) grade (junk bonds, high-yielding bonds)

**1.3 Fair pricing of straight bond**

fair price (fundamental price) of a bond is equal to the present value of the stream of the cash flow generated by the bond using a market-determined discount rate for bonds of the same maturity and the same risk class

**i) Annual discounting of annual coupon payments**

general case for a bond which pays coupon *m*-times per year and the discounting period is one *m*-th of the year (*m* = 2 is the most frequent case of semi-annual discounting of semi-annual coupons)

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| MATH NOTE |
| The pricing equation of a straight bond exploits the formula for the sum of a geometric series, which says that  We have  By substituting these values into the geometric series we obtain the sum of coupon payments in the bond pricing formula (annual discounting of annual coupon payments). |

**ii) Properties of price-yield relationship**

* the curve is downward sloping
* the curve is convex

*price-yield relationship*

* par bond

the equality of the coupon and discount rates implies the equality of the bond price and the principal (bond is selling at par)

a bond whose coupon rate is greater than the discount rate is priced above its par value (bond is selling with a premium)

a bond whose coupon rate is smaller than the discount rate is priced below its par value (bond is selling at a discount)

* the fair price of a perpetual bond

**iii) Valuation date is different from coupon payment date**

two-step discounting: 1. discounting to the date of the nearest coupon date and 2. discounting to the valuation date

price of the bond on the nearest coupon date

price of the bond on the valuation date

*saw-tooth development of the bond price between coupon payment dates (the case of par bond)*

**iv) Alternative pricing formulas**

* annual discounting of annual coupon payments
* semi-annual discounting of semi-annual coupon payments
* annual discounting of semi-annual coupon payments
* semi-annual discounting of annual coupon payments
* continuous discounting of annual coupon payments

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| MATH NOTE |
| Continuous discounting is the result of dividing the coupon period into infinitely many discounting sub-periods. If the coupon period has been divided into *n* discounting sub-periods, then the present value of a coupon obtained at the end of the first period is  If *n* approaches infinity, we get  Here we used the well-known property of Euler’s number. Knowing the factor for discounting back by one period, the present value of an amount received at the end of the period *t* is |

**1.4 Clean and dirty prices**

when a bond is transacted between coupon dates then the next paid coupon must be divided between the bond seller and the bond buyer proportionally to the number of days the bond has been held by both traders

**i) Division of coupon between bond seller and bond buyer**

coupon date (record date) is the date on which all registered holders of the bond are entitled to receive the coupon (paid by the issuer of the bond)

accrued coupon (accrued interest) is a proportion of a coupon earned from the beginning of the coupon period until the transaction day

clean price is the quoted price of the bond which excludes the accrued coupon

dirty price (full price) is the price which is actually paid for the bond; it consists of the clean price and the accrued coupon

ex-coupon date (ex-dividend date)is a special date situated several days before the coupon date; it plays the following role:

* when the bond is transacted before the ex-coupon date

the bond is traded *cum-dividend*, which means that the issuer will pay the entire coupon to the bond buyer (newly registered bond owner) who will compensate the bond seller for the accrued coupon

* when the bond is transacted on or after the ex-coupon date

the bond is traded *ex-dividend*, which means that the issuer will pay the entire coupon to the seller (original bond owner) who will compensate the buyer for the accrued coupon

*saw-tooth development of the dirty price around the clean price*

D

x

**ii) Analytical representation of dirty and clean prices**

the dirty price should be basically equal to the fair price of the bond at the transaction date because the buyer acquires the bond with a complete cash flow whose present value must be paid for

the dirty price is paid in two parts: as the quoted clean price and as the accrued coupon

when approaching the end of the coupon period, the dirty price declines and the accrued coupon increases so the clean price remains fairly stable over the coupon period

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| PROOF |
| Demonstration of the stability of the clean price over the coupon period assuming the case of a par bond.  A par bond is characterized by the equality between its price and nominal value on coupon dates which itself is the consequence of the equality between its coupon rate and the discount rate . We have  The adjustments to the dirty price formula use the following approximations that are justified if the numerical value of the interest rate is close to zero. |

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| EXAMPLE |
| A bond pays an annual coupon of 9%. A total of 302 days has passed since the last coupon date, and 63 days remain to the next coupon date (assuming 365 days in a year). At the beginning of the next coupon date the bond will have 5 years to maturity. The current market yield of the bond is 8%.  What are the dirty price and the clean price of the bond? |

**1.5 Yield to maturity**

yield to maturity (YTM) is defined as a discount rate at which the present value of the discounted cash flow of the bond is equal to the market price (dirty price) of the bond

YTM is the most popular measure of the rate of return from investing in bonds (other names are redemption yield, internal rate of return)

YTM in the case of the annual discounting of annual coupons is the solution of *T*-degree polynomial equation); YTM formula is identical to the equation that determines the price of the bond

**i) Meaning of YTM**

modification of the equation that defines the YTM

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* the right-hand side of the equation shows the terminal value of the accumulated cash flow from holding the bond with the assumption that all coupons can be reinvested at a constant YTM
* the left-hand side of the equation shows the terminal value at the time of maturity of the bond that results from investing an amount needed to buy the bond at an interest rate equal to YTM

YTM can be seen as a rate of interest on a one-off term deposit which replaces the strategy of receiving the coupons of the underlying bond at consecutive intervals and reinvesting them at the same rate of interest

**ii) Shortcomings of YTM**

* there is an implicit assumption that the bond is held to maturity
* there is an implicit assumption that all future coupons can be reinvested at a constant interest rate (disregarding the reinvestment risk)

**1.6 Other yield measures**

**i) Holding-period yield**

holding-period yield (HPY) is the average yield realized during the holding (investment) period which takes into account the purchase and the sale of the bond as well as changes in the rollover rates at which coupons can be reinvested

HPY is the solution of the equation

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... buying price of the bond

*…* selling price of the bond

*…* rollover (reinvestment) rate at the time *t*

if the HPY is used as a forward-looking indicator, assumptions about future values that are uncertain must be made

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| EXAMPLE |
| Determine the holding-period yield of a bond which was purchased on a coupon date at a price of 96.5 and sold exactly four two later at 101.3. The bond paid a coupon of 8.7% with semi-annual frequency. The rollover rates for the first three coupons were 10%, 10.2% and 10.4 % respectively.  The HPY is the solution to the equation  We have |
|  |

**ii) Current yield**

current yield (CY) is the yield to maturity of a perpetual bond (other names are flat yield, running yield, interest yield, income yield)

CY is a satisfactory approximation of the YTM for long-dated bonds

CY uses a more stable clean price even though the dirty price would be more appropriate because it is the purchasing price of the bond

CY does not take into account capital gains or losses arising from the differences between the purchasing price of the bond and its maturity value

**iii) Simple yield to maturity**

simple yield to maturity (SYM) is the current yield adjusted for capital gain or loss which arise from holding the bond and which are spread out evenly over the holding horizon

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| EXAMPLE |
| A bond pays an annual coupon of €8.75 and its current clean price is €95.30. The bond has exactly four years to maturity. What is the bond’s current yield, simple yield to maturity and yield to maturity?  With the help of the Excel function YIELD one can get |

**iv) Money market yield**

yields of bonds maturing in less than one year should be directly comparable with yields of money market instruments (i.e. certificates of deposits)

money market yield (MMY) is the YTM calculated with the use of money market conventions (simple interest instead of compound interest, actual number of days instead of fixed number of days in coupon period)

* bond matures in less than half a year (bond pays only last coupon at maturity)

capital market conventions money market conventions

* bond matures in more than half a year (bond pays the penultimate coupon a half year before maturity and last coupon at maturity)

capital market conventions

money market conventions

**1.7 Price-maturity relationship**

the clean price of a bond may change simply because the bond is heading to maturity

* coupon rate is higher than yield ()

bond is selling at a premium

bond’s clean price converges to its nominal value from above

* coupon rate is smaller than yield ()

bond is selling at a discount

*price-maturity relationship*

*T*

premium bond

discount bond

par bond

**2 ANALYSIS OF YIELD CURVE**

yield curve(term structure of interest rates) is the relationship between a measure of a bond’s yield (most frequently yield to maturity) and a bond’s time to maturity

a yield curve should be constructed from a homogeneous group of bonds (the same market segment, the same risk class)

*shapes of yield curves*

YTM

time to maturity

with a hump

declining (reverted)

rising

flat

practical problems in assembling an empirical yield curve

* missing maturities in selected given classes of bonds
* more coupon-bearing bonds with the same maturity but different yields
* weaknesses of the YTM measure itself (unawareness of reinvestment risk, holding bond till maturity)

**2.1 Zero yield curve**

zero-coupon bond (pure discount bond) is a bond which pays no explicit coupon and whose cash flow consists of only two components: purchasing price and principal amount received at maturity

interest on zeros is earned indirectly by way of purchasing the bond at a discount from its nominal value advantage of zeros consists of the absence of reinvestment risk

*t*-year zero yieldis the yield to maturity of *t*-year zero-coupon bond

zero yield curveis a yield curve constructed from the yields of zero-coupon bonds

**i) Bootstrapping**

bond stripping (unbundling) is a technique of financial engineering that breaks down the coupon-bearing bond into a series of zero-coupon bonds that are traded separately

the cash flow from individual zero-coupon bonds is backed by the cash flow of the underlying coupon-bearing bond

C

C

C

C

M

C

C

C

C

M

Coupon

bond

Zero 1

Zero 2

Zero T

Zero3

Zero M

cash flow from the underlying coupon bond

cash flow from the pool of zero-coupon bonds

bootstrapping is an analytical technique that derives the yields of zero-coupon bonds from a given set of coupon-bearing bonds

is a set of observed yields of coupon-bearing bonds whose prices are and coupons are

is a set of yields of zero-coupon bonds that are consistent with the given set of coupon-bearing bonds

step 1: determination of one-year zero yield

a one-year bond is expected to be a zero-coupon bond, therefore

step 2: determination of a two-year zero yield

the present value of the two-year coupon-bearing bond

the present value of one-year and two-year zero-coupon bonds that were stripped from the two-year coupon-bearing bond

since the two cash flows are identical, no arbitrage condition requires that their present values are identical too

this is an equation that can be solved for the unknown two-year zero rate

step 3: in a similar way, all remaining zero yields can be consecutively calculated one by one, beginning with the lowest maturities and moving to the highest

there is no one-to-one correspondence between observed market yields of coupon-bearing bonds and zero rates; a different set of coupon bonds, even with the same yields, would result in a different set of zero rates

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| EXAMPLE |
| Calculate zero rates consistent with a given set of coupon bonds using the bootstrapping method.   |  |  |  |  | | --- | --- | --- | --- | | Bond | Price  (€) | Coupon  (%) | Maturity  (years) | | A | 90.91 | 0 | 1 | | B | 97.41 | 9 | 2 | | C | 85.26 | 5 | 3 | | D | 104.65 | 13 | 4 |   Determination of a one-year zero yield.  Determination of a two-year zero yield.  Determination of a three-year zero yield.  Determination of a four-year zero yield. |

**ii) Synthetization**

synthetic security is a bundle of securities whose combined cash flow is the same as the cash flow of the imitated security

synthetic zero-coupon bond is a set of coupon-bearing bonds whose combined cash flow creates the cash flow of the zero-coupon bond

synthetization is an analytical technique which imitates a zero-coupon bond with an appropriate combination of coupon-bearing bonds; a synthetic bundle of coupon bonds must eliminate all coupons of the original bond except for the final one

cash flow from original coupon-bearing bond

cash flow from synthetic set of coupon-bearing bonds

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| EXAMPLE |
| Calculate zero rates consistent with a given set of coupon bonds using the synthetization method.   |  |  |  |  | | --- | --- | --- | --- | | Bond | Price  (€) | Coupon  (%) | Maturity  (years) | | A | 90.91 | 0 | 1 | | B | 97.41 | 9 | 2 | | C | 85.26 | 5 | 3 | | D | 104.65 | 13 | 4 |   Determination of a one-year zero yield.  Determination of a two-year zero yield.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Year | 1B | 1A | mA | 1B+mA | | 0 | -97.41 | -90.91 | 8.182 | -89.228 | | 1 | 9 | 100 | -9 | 0 | | 2 | 109 |  |  | 109 | | multiplier |  |  | -0.090 |  |   The multiplier mA represents a number of A bonds that eliminate the first year’s combined net cash flow (composed of one long B bond and mA short A bonds.  Determination of a three-year zero yield.   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Year | 1C | 1B | mB | C+mB | 1A | mA | C+mB+mA | | 0 | -85.26 | -97.41 | 4.481 | -80.779 | -90.91 | 4.173 | -76.606 | | 1 | 5 | 9 | -0.414 | 4.586 | 100 | -4.59 | 0 | | 2 | 5 | 109 | -5 | 0 |  |  | 0 | | 3 | 105 |  |  | 105 |  |  | 105 | | multiplier |  |  | -0.046 |  |  | -0.0459 |  |   The multiplier mB represents a number of B bonds that eliminate the second-year’s combined cash flow (composed of one long C bond and mB short B bonds).  The multiplier in column mA represents a number of A bonds that eliminate the first-year’s combined cash flow (composed of one long C bond, mB short B bonds and mA short A bonds).  Determination of four-year zero yield.  Continuing with this problem is voluntary. The result must be the same as in the problem using the bootstrapping method. One should obtain |

**2.2 Implied forward yield curves**

spot zero rate is the rate charged for immediate borrowing and lending

current *p*-year zero rate

forward zero rate(forward-forward)is the rate charged (or expected) now for future borrowing and lending

*p*-year forward zero rate which starts at time *t* (at the beginning of the period *t*+1) and ends at time *t*+*p* (at the end of the period *t*+*p*)

*T*-th period

*t*-th period

2nd period

1st period

forward rates must not be confused with future spot rates (they may be used as a best estimate of future interest rates)

*p*-year zero future rate that will exist at time *t* (at the beginning of the period *t*+1)

spot rates can be seen as a special case of future and forward rates

a complete description of forward rates would need to highlight when negotiations take place or expectations are formed

*p*-year forward rate which will start at time *t*, will end at time *t*+*p* and was expected or negotiated at time *e* (if *e* = 0 then the superscript can be omitted)

**i) Implied forward rates**

implied forward rates are forward rates that are consistent with a given spot zero yield curve

in efficient financial markets a yield on an investment over a given time period should not depend on an adopted reinvestment strategy (inconsistent implied forward rates would indicate the existence of an arbitrage opportunity)

the yield from purchasing two-year zero-coupon bonds in a monetary amount of *A*

the yield from purchasing one-year zero-coupon bonds in a monetary amount of *A* and rolling over the investment for another year by again buying zero-coupon bonds

the consistency condition requires the equality between the two cash flows from which we have

general formula

spot yield is the geometric mean of forward yields (using convention)

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| EXAMPLE |
| The following spot zero rates were extracted from a given structure of coupon bond yields (they are borrowed from the previous example):   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Maturity | 1 | 2 | 3 | 4 | | Spot rate | 10.00 | 10.52 | 11.08 | 11.66 |   Calculate the forward rates and then verify the validity of the equation  The two-year forward rate starting in one year from now  The one-year forward rate starting in three years from now  Checking validity: |

**ii) Synthetic forward rates**

synthetic forward rates are forward rates that are created by an appropriate combination of borrowing and lending at zero spot rates

termination of (*t*+*p*)-year lending

termination of *t*-year borrowing

lending for *t*+*p* years

borrowing for *t* years

structure of combined (synthetic) cash flow

time 0: [cash inflow: borrowing for *t* years at ]

[cash outflow: lending for *t*+*p* years at ]

time *t*: [cash outflow: termination of *t*-year borrowing]

time *t*+*p*: [cash inflow: termination of (*t*+*p*)-year lending]

the above combination of spot borrowing and lending effectively creates a *p*-year forward loan (starting at time *t* and ending at time *t*+*p*) with a lock-in forward rate

**iii) Forward yield curves**

forward rates can be arranged in forward yield curves

forward yield curve expected in one year’s time

forward yield curve expected in two years’ time

forward yield curve expected in three years’ time

waning property: the more distant the time location of the forward yield curve, the shorter its range of values

*family of implied forward yield curves*

*T*-2

forward yield curve in one year’s time

zero yield curve

*T*-1

*T*

3

2

1

forward yield curve in two years’ time

calculation of *T*-year forward expected in one year from now would require the (*T*+1)-year spot rate which is, however, not available, so the last calculable forward rate is

position property

* an upward sloping zero yield curve is below all forward yield curves (all *p*-year forward rates are higher than the current *p*-year rate)

for all *t*

* a downward sloping zero yield curve is above all forward yield curves (all *p*-year forward rates are smaller than the current *p*-year rate)

for all *t*

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| PROOF |
| Validation of the position property follows from the consistency condition between spot and forward rates. Assuming a rising yield curve, we have  because After applying the *p*-root to the above inequality, we have    This shows that each *p*-year forward rate is higher than the current *p*-year rate.  Specifically, for *t* = *p* = 1 we have that a one-year rate in one year from now is expected to be higher than the current one-year rate. |

**iv) Expectation hypothesis**

implied forward rates are the best indicator of expected future interest rates

… *p*-year implied forward rate starting in *t* years’ time that is expected now

… *p*-year spot rate that will exist in *t* years from now

*E* … expectation operator

current spot rates may change under the pressure of changing expectations (confirmation of the constituency conditions between the spot and forward rates)

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| EXAMPLE |
| The current one-year interest rate is 10%. The market expects that a one-year interest rate in one year from now will be 11.05% and a one-year interest rate two years from now will be 12.18%. What is the three-year interest rate?  The consistency between spot and forward rates requires that  from which we derive |

**2.3 Pricing of floating rate notes**

floating rate note (FRN) is a type of bond whose coupon rate is fixed for a given period by reference to some short-term market interest rate (for example three-month LIBOR, six-month LIBOR) and reset periodically on the coupon reset dates

**i) Par property**

the expectation hypothesis ensures that the fair price of a floater on a coupon payment date is equal to the nominal value of the bond

the expectation hypothesis is supported by the evidence showing that FRN are priced at or near par value

… coupon paid at the end of the period *t* which is linked to a one-year money market interest rate prevailing at the beginning of the period *t* whose best predictor is the implied one-year forward rate expected at the beginning of period *t* (in (*t*-1) years from now)

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| PROOF |
| The par property follows from the consistency condition between spot and forward rates.  A valuation formula assumes that coupons are linked to the future one-year money market rates whose best predictor is implied forward rates. |

**ii) Synthetic floater**

the cash flow from a floater can be reproduced synthetically by a sequence of investing in a short-term money market instrument and reinvesting the principal on a rolling basis

*total cash flow from synthetic floater strategy*

*net cash flow from synthetic floater strategy*

fair price of synthetic floater strategy

price of the synthetic strategy one year before maturity

price of the synthetic strategy two years before maturity

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

price of the synthetic strategy at the beginning of the investment period

**2.4 Inflation-linked bond**

inflation-linked bond(inflation-indexedbond) is a bond whose cash flow (coupons and principal repayment) takes into account the evolution of a particular price index with the aim to protect the bond holder against inflation

bonds are issued with a certain real coupon (i.e. 2 – 2.5 %) which is indexed by the rate of inflation

the ratio of two price indexes determines the rate of inflation over the period in question

**i) Reasons for imperfect protection against inflation**

the weighted composition of an applied price index need not coincide with the collection of goods consumed by investors who want to be protected against inflation

price indexes, which are used for the purpose of calculating the cash flow from the bond, are to some extent outdated compared to price indexes valid at the time of receiving the cash flow from the bond

* a delay of up to two months caused by lags in statistical reporting and processing
* a delay of up to six months caused by the need to fix the size of the coupon at the beginning of the coupon period because of the determination of the accrued coupon

**ii) Nominal and real yield to maturity**

nominal YTM for *T*-year inflation-linked bond

real YTM for *T*-year inflation-linked bond

Fisher equation

or

… nominal yield, … real yield, … inflation rate

equivalence of nominal and real pricing formulas

**iii) Break-even inflation**

break-even inflation is the rate of inflation that makes the nominal yield on an inflation-linked bond equal to the yield on a conventional bond with the same maturity

break-even inflation can be calculated using the Fisher equation (nominal yield on conventional bond and real yield on inflation-linked bond are observable variables)

break-even inflation makes observable the expected inflation on efficient markets (a difference between the two inflations would activate the adjustment process until equality was restored)

* the expected inflation is higher than the break-even inflation

⇒ investors would prefer an inflation-linked bond over a conventional one because coupons of inflation-linked bond will be indexed to the actual inflation, which is expected to be higher than the inflation which makes an inflation-linked bond as attractive as a conventional bond

⇒ a higher demand for an inflation-linked bond drives its price up

⇒ a higher price of an inflation-linked bond pushes its real yield to maturity down

⇒ a lower real yield on an inflation-linked bond implies that the calculated value of break-even inflation is higher and closer to the expected inflation

⇒ the process continues until the break-even inflation converges to the level of the expected inflation

* the expected inflation is lower than the break-even inflation

⇒ an opposite adjustment process is put in motion which will not be completed until the break-even inflation converges with the level of the expected inflation

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| EXAMPLE |
| A conventional five-year Treasury bond has a nominal yield of 8.3% and an inflation-linked Treasury bond of the same maturity has a real yield of 2.5%. Both bonds pay coupons semi-annually. What is the annual break-even inflation?  This is the Fisher equation for bonds paying coupons with semi-annual frequency:  Solving this equation for break-even inflation, we get |

**2.5 Par yield curve**

par bond is a bond that is priced at or near par (price of the bond is equal to its nominal value)

par yield is the yield to maturity of a par bond

par yield is equal to the coupon rate of par bonds (if the coupon rate and YTM are equal, then the bond price and its nominal value will also be equal)

par yields are used to determine the required coupon on new bonds that are to be issued at par

par yield curve is a yield curve constructed from par yields

consistency between par yields and zero yields

* calculation of par yields from given zero yields

zero yields determine discount factor which are used to calculate par yields using these formulas

* calculation of zero yields from given par yields

par yields are used to calculate discount factors , using these formulas

and the discount factors determine zero yields, using this formula

|  |
| --- |
| EXAMPLE |
| Calculate par yields from the given set of zero yields.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Maturity | 1 | 2 | 3 | 4 | | Zero rate | 10.00 | 10.52 | 11.08 | 11.66 | | Discount factor | 0.909091 | 0.818688 | 0.729613 | 0.643229 | |

**i) Position property**

* an upward sloping spot zero yield curve is situated above the par yield curve
* a downward sloping spot zero yield curve is situated below the par yield curve

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| PROOF |
| Assuming an upward sloping spot zero yield curve, we have  An analogous method can be used in the case of a downward sloping spot zero yield curve. |

*joint position property of spot, forward and par yields*

forward yields

spot yields

par yields

par yields

spot yields

forward yields

**3 MEASURING INTEREST RATE RISK**

**3.1 Risks associated with holding bonds**

* interest rate risk (price risk, market risk) is the risk of a change in the bond’s price as a result of a change in the market yields (by changing the price, the yield of the bond adjusts to prevailing market conditions)

this risk is irrelevant for investors who hold the bond to maturity

* reinvestment risk is the risk that a bond coupon will have to be reinvested at a lower interest rate, which reduces the yield to maturity

the risk is less urgent for floaters and irrelevant for zero-coupon bonds

* inflation risk is the risk that the purchasing power of coupons and principal received in the future will be eroded by a higher inflation than initially expected

the risk is less relevant for inflation-linked bonds

* call risk is the risk that the issuer will retire the bond before its scheduled maturity by exercising the call provision in the callable bond contract

as bonds are called mainly in response to a decline in market yields, the bondholders are able to reinvest proceeds from earlier sales at lower interest rates

the capital gain which results from declining bond prices is capped by the given call price stipulated in the bond contract

the cash flow from holding the bond is not known with certainty

* liquidity risk (marketability risk) is the risk that the ease of trading the bond near the prevailing market price will be impaired

the higher the liquidity risk the wider the bid-ask spread quoted by bond dealers (the difference between the bid price at which the dealer is prepared to buy the security and the ask price at which the dealer is prepared to sell the security)

the wider the bid-ask spread, the costlier the trading of the bond

* credit risk is the risk of a change in the bond’s price associated with the occurrence of a credit event

typical credit events:

*default* - inability of the issuer of the bond to honour his/her contractual obligations (payments of coupons and/or principal)

*downgrade* - a cut in rating by a rating agency based on the deteriorating earning capacity to honour obligations from the bond

*flight to quality* - liquidation of bond holdings due to broader economic factors (worsened macroeconomic outlook, danger of political instability, imminent financial turbulence, contagion and others)

* exchange rate risk (currency risk) arises when the issuer makes payments in a foreign currency so the cash flow from the bond in terms of domestic currency depends on the foreign exchange rate at the time the cash flow is received

**3.2 Duration**

two similar measures of duration are in use: Macaulay duration and modified duration (the term duration mostly refers to the Macaulay duration)

**i) Macaulay duration**

Macaulay duration is defined as the weighted average of the times in which the cash flow from the bond is received (average time of receiving the bond’s cash flow)

weighted variable: time of obtaining the cash flow

the unit of measurement of the Macaulay duration is one year (more generally a time period unit)

weights: the present values of an individual cash flows relative to the bond price

the pricing formula for straight bonds ensures that the sum of all weights is equal to one

sum formula of the Macaulay duration

the Macaulay duration for a bond which pays coupon *m*-times per year and the discounting period is one *m*-th of the year

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| --- |
| EXAMPLE |
| A bond with a yield to maturity of 8% and a coupon of 5% paid annually has exactly five years to maturity. The nominal value of the bond is 100. Calculate the Macaulay duration.  The price of the bond is  For the Macaulay duration we have  The validity of these results can be checked by using the sum formulas: |

|  |
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| MATH NOTE |
| The sum formula of duration can be derived using the formula for the sum of the arithmetico-geometric sequence:  We have  so after several adjustments we get  Using this intermediate result, the complete sum formula of the Macaulay duration can be written as |

**ii) Modified duration**

modified duration is the change in the bond price with respect to the change in the yield, expressed as a percentage of the bond price (the minus sign ensures that MoD is a positive number)

the unit of measurement of the modified duration is per cent (as opposed to a Macaulay duration which is measured in time units)

relationship between MaD and MoD

both measures of duration are identical when using continuous discounting

**3.3 Properties of Macaulay duration**

**i) Limiting value**

for coupon bonds, duration is always less than time to maturity because some weight is given to cash flows received in earlier coupon payment dates (the average time of obtaining the cash flow is less than the bond’s time to maturity)

for zero-coupon bonds, the duration is equal to the time to maturity because the entire cash flow occurs at the time of the maturity of the bond

duration of perpetual bond ()

**ii) Measure of interest rate risk**

the duration is the elasticity (sensitivity) of the bond price with respect to the change in the market yield

the duration is also the logarithmic derivation of the bond’s price with respect to the bond’s yield

*logarithmic derivation*

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| MATH NOTE |
| Logarithmic derivation is another manner of expressing the indicator of elasticity which measures responsiveness of an explained variable to a change in an explanatory variable. Specifically, the elasticity *ε* is defined as the ratio of the percentage change in an explained variable to the percentage change in an explanatory variable,  where *y* is the explained variable, *x* is the explanatory variable and is the derivation of the function between variables *y* and *x*.  We can write  By substituting these expressions into the elasticity formula, we get |

the duration can be used as the first order approximation of a change in the bond price which results from a change in the bond yield

as yields fall, the price of a bond with a low duration rises less than the price of a bond with a high duration and vice versa

the greater the change in the bond yield, the greater the approximation error

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| --- |
| EXAMPLE |
| You have the bond from the previous example: the yield to maturity is 8%, the annual coupon is 5%, the nominal value is 100, the current bond price is 88.02, the time to maturity is 5 years and the duration is 4.51 years. Approximate a decrease in the bond price that would result from the rise in the market yield by two percentage points.  Using the approximation formula, we have  The bond price is expected to fall from 88.02 to 80.67 (= 88.02 – 7.35). An exact new bond price, calculated by using the bond valuation formula, is 81.05. Therefore, an exact fall in the bond price is 6.97. The approximation property of duration thus underestimates the true new bond price by 4.7%. |

**iii) Immunization property**

an investment strategy of holding and selling a coupon bond prior to maturity entails two mutually offsetting effects caused by a change in market yields

* as interest rates rise, the value of the bond declines and the return from reinvested coupons increases
* as interest rates fall, the value of the bond increases and the return from reinvested coupons declines

the immunization property maintains that the two effects offset each other at a point of time which is equal to the bond’s duration

the immunization hedging strategy involves the construction of a bond portfolio that has an assured return over a given horizon (composed of reinvested coupons and values of bonds)

* the market value of the bond portfolio is equal to the present value of a hedged liability
* the duration of the bond portfolio is equal to the time horizon at the end of which the liability should be met

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| PROOF |
| The letter *A* denotes the total value of reinvested coupons accumulated at a specified point of time *X*  Derivation of *A* with respect to *r* results in  The letter *B* denotes the value of the bond at the same point of time *X* (it is equal to the discounted value of the remaining cash flow from the bond):  Derivation of *B* with respect to *r* results in  This equation requires that at the point of time *X* the change in the reinvested coupons is exactly offset by the change in the bond price,  After substituting in this equation and making a few adjustments one can obtain this expression  The contents in the first parentheses are the pricing equation of a coupon bond,  Therefore  from which we get  We used the analytical formula derived earlier for the Macaulay duration. |

limitations of the immunisation strategy

* restrictive assumptions about a change in interest rates (parallel shift of a horizontal yield curve immediately after the immunized position was created)
* protection is provided only against small interest rate changes (duration is the first order approximation of interest rate changes)
* time decay means that the passage of time reduces the duration more slowly relative to the shortening of the hedging horizon

*time decay*

duration

time to maturity

time

*H*

*H* < *D*

*H* = *D*

*H - t*

**iv) Duration of a bond portfolio**

the duration of a bond portfolio is equal to the weighted average of durations of individual bonds, using relative market values of bonds as weights

… market value of all bonds

**Measuring interest rate and credit risk**

**7**

*end of hedging horizon*

*years*

… duration of the bond

|  |
| --- |
| PROOF |
| Recall the relationship between the Macaulay and the modified durations,  Without the loss in general validity we assume that the portfolio is composed of two bonds. We can write |
|  |

**3.4 Convexity**

convexity is a measure of the second-order (quadratic) approximation of a change in the bond price which results from a change in the bond yield (in contrast to duration as a measure of the first-order or linear approximation of sensitivity of the bond price to the bond yield)

convexity measures the curvature of the price-yield relationship (in contrast to duration as a measure of the slope of the price-yield relationship)

**i) Convexity formula**

convexity is the second-order derivative of the bond price with respect to the bond yield, relative to the bond price

**ii) Approximation property**

the application of the first two terms of the Taylor series on the price-yield relationship results in the following

the convexity term is always a positive number in contrast to the duration term which may become either a positive or negative number

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| EXAMPLE |
| You have the bond from the previous example: the yield to maturity is 8%, the annual coupon is 5%, the nominal value is 100, the current bond price is 88.02, the time to maturity 5 years and the duration is 4.51 years. Approximate the decrease in the bond price that would result from the rise of the market yield by two percentage points, using both duration and convexity terms.  Calculation of convexity  Approximation of a change in the bond price gives  The bond price is expected to fall from 88.02 to 81.06 (= 88.02 – 6.96). We already know that the new bond price is exactly 81.05. By adding the convexity term the approximation error in predicting the new bond price has been thus reduced from 4.7% to only 0.012%. |

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| MATH NOTE |
| The Taylor series which is one of the central tools in mathematical analysis provides a representation of a function around a given point by a Taylor polynomial. If is infinitely differentiable at point , we have  A function can be approximated by using a finite number of terms of its Taylor series. An approximation of a change in the bond price by using the duration term and the convexity term uses the first two Taylor polynomials,  where the function is the bond pricing formula is the initial yield to maturity, and is a new yield to maturity. To rewrite this expansion in terms of known bond-yield notation, we have to do following substitutions  We get |

**iii) Attractiveness of convexity**

when two bonds have the same price, the same yield and the same duration (the same slope of the price-yield relationship), the more convex bond of the two is more attractive from the investor’s point of view

* when the yield increases, the price of more convex bond falls less than the price of less convex bond
* when the yield declines, the price of a more convex bond rises more than the price of less convex bond

*attractiveness of convexity*













*ln* (*1+r*)

*ln P*

*D*

such a situation, however, is not likely to last for a long time

the higher attractiveness of a more convex bond which outperforms a less convex bond under all circumstances would result in a higher price and a lower yield of the more convex bond

**4 MEASURING CREDIT RISK**

credit risk is related to potential losses for bondholders that may arise from the occurrence of an event which impairs the capacity of bond issuers to fulfil their contractual obligations

typical credit events

* default – the issuer of the bond is not able to honour his/her contractual obligations (payments of coupons and/or principal)
* downgrade – a recognized rating agency lowers the credit rating of the bond based on an evaluation of the debtor’s earning capacity to honour obligations from the bond
* flight to quality – liquidation of bond holdings due to wider economic factors (worsened macroeconomic outlook, danger of political instability, imminent financial turbulence, contagion and others)

*credit rating systems of established rating agencies*

Default

Speculative

grade

Investment

grade

|  |  |  |
| --- | --- | --- |
| **Moody’s:** | **Standard & Poor‘s** | **Fitch** |
| Aaa | AAA | AAA |
| Aa1, Aa2, Aa3 | AA+, AA, AA- | AA+, AA, AA– |
| A1, A2, A3 | A+, A, A– | A+, A, A– |
| Baa1, Baa2, Baa3 | BBB+, BBB, BBB– | BBB+, BBB, BBB– |
| Ba1, Ba2, Ba3 | BB+, BB, BB– | BB+, BB, BB– |
| B1, B2, B3 | B+, B, B– | B+, B, B– |
| Caa1, Caa2, Caa3 | CCC+, CCC, CCC– | CCC+, CCC, CCC– |
| Ca | CC | CC |
| C | C | C |
|  | RD, SD, D | DDD, DD, D |

**4.1 Credit spread**

credit spread is the excess of the yield of the risky bond over the yield of the risk-free bond

credit spread is an indication of the risk premium that compensates bondholders for a higher exposure to credit risk (compensation for eventual losses from credit events)

other factors may also influence the spread (liquidity of the bond, inflation risk etc.)

credit spread notation

zero rates of risk-free bonds (extracted from risk-free AAA government bond yields)

zero rates of bonds for a given credit rating X (extracted from risky coupon bonds of the rating X)

X = AA, A, BBB, . . .

credit spreads of bonds for a given credit rating X

credit yield curves are yield curves constructed from bonds of a given credit rating

*family of credit yield curves*

maturity

yield

AAA

AA

BBB

A

empirical regularities

* credit yield curves corresponding to lower credit evaluations are situated above the credit yield curves corresponding to higher credit evaluations (holder of riskier bonds should be compensated by a higher yield)
* credit yield curves are upward sloping (a bond with a longer time to maturity is more likely to default than a bond with a shorter time to maturity)
* credit yield curves corresponding to a lower credit rating are steeper than credit yield curves corresponding to a higher credit raging (the previous regularity is more pronounced for lower grade yield curves)

**4.2 Risk-neutral probabilities of default**

expected value of a random variable is the probability-weighted average of all possible outcomes of this variable

EV = (probability of *n*-th outcome) × (value of *n*-th outcome)

objective certainty means that the law of large numbers ensures that the expected value is the sure average outcome if a risky game is repeated many times

subjective certainty means that the expected value of a one-off experiment is subjectively regarded as a sure event

properties of a risk-neutral environment

* investments earning the expected value (sure outcome) should generate risk-free returns
* the expected value of the investment when discounted at a risk-free rate of return should be equal to the current value of the investment

risk-neutral probabilities are theoretical probabilities consistent with the risk-neutral environment (as opposed to empirical probabilities)

simple model of risk-neutral probabilities of default

expected pay-off from the *T*-year zero-coupon bond at maturity

probability that the risky bond will default at maturity

… recovery rate (proportion of the principal received in the event of default)

valuation bond formulas

… price of risky bond

… price of risk-free bond

no arbitrage condition between two valuations of the bond

|  |
| --- |
| EXAMPLE |
| A ten-year BBB corporate bond has a spread of 220 basis points over a ten-year Treasury bond. Historical recovery rate for bonds of this risky class is 40%. An approximate value of risk-neutral probability that the bond will default at maturity is |

loss given default (LGD) is an estimation of the percentage loss in the event of default

LGD = 1 – *R*

The expected loss from holding a risky bond

|  |
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| EXAMPLE |
| What is the expected percentage loss from holding a three-year zero-coupon corporate bond with a yield of 5.7% if you know that a risk-free government bond yields 5%? |

**4.3 Historical probabilities of default**

historical (empirical) probabilities of default are based on rich data sets of bond defaults throughout their economic life

marginal mortality rate (MMR) is the probability that a bond of a credit rating *X* will default in a given year of its life

definitive MMRs are obtained as weighted average of a sample of MMRs calculated in individual years

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Rating** | **Maturity** | | | | | |
| **1** | **2** | **3** | **…** | **…** | **15** |
| **AAA** | 0.00 | 0.00 | 0.04 | **…** | **…** | 0.67 |
| **AA** | 0.01 | 0.04 | 0.10 | **…** | **…** | 1.39 |
| **A** | 0.04 | 0.12 | 0.21 | **…** | **…** | 2.59 |
| **BBB** | 0.24 | 0.55 | **0.89** | **…** | **…** | 6.85 |
| **…** | **…** | **…** | **…** | **…** | **…** | **…** |
| **CCC** | 25.26 | 34.79 | 42.16 | **…** | **…** | 66.12 |

is the empirical probability that the bond issued with the credit rating BBB will default in the third year of its life

historical probabilities of default tend to be significantly lower than risk-neutral probabilities of default because only a part of the spread can be associated with the default risk

investors want to be compensated for other risks as well, such as market risk, liquidity risk, inflation risk, unforeseen bad scenarios, etc.

survival rate (SR) is the probability that a bond of a credit rating X will not default in a given year of its life

cumulative mortality rate (CMR) is the probability that a bond of a credit rating X will default over a given period *T*

**5 MORTGAGES**

mortgages are various types of long-term financial instruments that intermediate housing financing

mortgage market is a collection of markets that include a primary (origination) market where funds are borrowed for financing housing and a secondary market in which mortgages trade

mortgage-backed securities (MBS) represent various types of securities that derive their cash flow from an underlying pool of mortgages

**5.1 Level-payment mortgage**

balloon mortgage has the same cash flow as a straight bond, which means that instalments cover only interest payments, and the original balance is repaid at the maturity of the mortgage

repayment of the entire balance at maturity may push mortgage debtors into serious troubles (a high number of defaults during the Great Depression in 1930s gave rise to the innovation of level-payment mortgages)

level-payment mortgage (traditional, annuity, plain-vanilla mortgage) is characterized by equally sized instalments throughout the entire life of the mortgage

instalments are made up of an interest payment based on a fixed mortgage rate and a repayment of part of the original mortgage balance

mortgage loan

regular mortgage instalment (annuity)

mortgage rate

term of mortgage

**i) Annuity formula**

the principle of fair pricing requires that the mortgage loan and the present value of all instalments discounted at a given mortgage rate are equal (we use the same formula as for the sum of coupon payments of a straight bond)

this relationship can be rearranged with the aim to determine the annuity level

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| EXAMPLE |
| A bank extended a 30-year level-payment mortgage in an amount of 100,000 USD at an annual mortgage rate of 9.5%. The mortgage is repaid in monthly instalments. What is the amount of the regular payments?  We have  By substituting the above numbers into the annuity formula we get |

**ii) Decomposition of instalment into interest payment and principal repayment**

in each period a fixed mortgage payment consists of the interest paid on an outstanding mortgage balance and the repayment of a portion of the outstanding mortgage balance

… mortgage balance outstanding at the end of period *t*

this relationship is a version of the difference equation that can be solved for the mortgage balance outstanding at the end of period *t*

formula for the interest payment in period *t*

formula for the principal repayment in period *t*

|  |
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| MATH NOTE |
| The decomposition of a regular instalment into interest payment and principal repayment represents a simple case of a linear difference equation of the first order with constant coefficients. This type of difference equation can be written as    whose general solution is  Let us rewrite the decomposition formula along the lines of the above difference equation. We have  By comparing their coefficients, we see that  We can now replace coefficients in the general solution with their counterparts in the decomposition formula. We get |

**iii) Payment calendar – annuity approach**

payment calendar is a table which arranges the main components of the mortgage’s cash flow

the annuity approach is based on the calculation of the annuity over the whole term of the mortgage

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time | Beginning  balance | Regular  instalment | Interest payment | Principal  repayment | Unpaid balance |
| 1 |  | A |  |  |  |
| 2 |  | A |  |  |  |
| . . . | . . . . . | . . . . . | . . . . . | . . . . . | . . . . . |
| T |  | A |  |  | 0 |

compilation of payment calendar

* fill the column *Regular instalment* with the annuity level calculated by using the annuity formula
* calculate the *Interest payment* in the first row as a product of the beginning balance and the mortgage rate
* calculate the *Principal repayment* in the first row by deducting the interest payment from the beginning balance
* calculate the *Unpaid balance* in the first row by deducting the principal repayment from the beginning balance
* the *Unpaid balance* in the firsts row becomes the *Beginning balance* in the second row
* fill all remaining rows in the same way

consistency checks

* the portion of the interest payment gradually decreases and the portion of the principal repayment gradually increases
* the unpaid balance in the last row must be zero
* the sum of all principal repayments must be equal to the beginning balance in the first row

|  |
| --- |
| EXAMPLE |
| Make out the payment calendar for a mortgage with a principal of 200,000 USD, a time to maturity of 20 years and an annual mortgage rate of 3%. Suppose that the mortgage is repaid in annual instalments.  The size of the annuity (regular instalment) is   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **Year** | **Beginning**  **balance** | **Regular**  **instalment** | **Interest**  **payment** | **Principal repayment** | **Unpaid**  **balance** | | 1 | 200,000.00 | 13,443.14 | 6,000.00 | 7,443.14 | 192,556.86 | | 2 | 192,556.86 | 13,443.14 | 5,776.71 | 7,666.44 | 184,890.42 | | 3 | 184,890.42 | 13,443.14 | 5,546.71 | 7,896.43 | 176,993.99 | | 4 | 176,993.99 | 13,443.14 | 5,309.82 | 8,133.32 | 168,860.67 | | 5 | 168,860.67 | 13,443.14 | 5,065.82 | 8,377.32 | 160,483.35 | | 6 | 160,483.35 | 13,443.14 | 4,814.50 | 8,628.64 | 151,854.71 | | 7 | 151,854.71 | 13,443.14 | 4,555.64 | 8,887.50 | 142,967.21 | | 8 | 142,967.21 | 13,443.14 | 4,289.02 | 9,154.13 | 133,813.08 | | 9 | 133,813.08 | 13,443.14 | 4,014.39 | 9,428.75 | 124,384.34 | | 10 | 124,384.34 | 13,443.14 | 3,731.53 | 9,711.61 | 114,672.72 | | 11 | 114,672.72 | 13,443.14 | 3,440.18 | 10,002.96 | 104,669.76 | | 12 | 104,669.76 | 13,443.14 | 3,140.09 | 10,303.05 | 94,366.72 | | 13 | 94,366.72 | 13,443.14 | 2,831.00 | 10,612.14 | 83,754.58 | | 14 | 83,754.58 | 13,443.14 | 2,512.64 | 10,930.50 | 72,824.07 | | 15 | 72,824.07 | 13,443.14 | 2,184.72 | 11,258.42 | 61,565.65 | | 16 | 61,565.65 | 13,443.14 | 1,846.97 | 11,596.17 | 49,969.48 | | 17 | 49,969.48 | 13,443.14 | 1,499.08 | 11,944.06 | 38,025.42 | | 18 | 38,025.42 | 13,443.14 | 1,140.76 | 12,302.38 | 25,723.04 | | 19 | 25,723.04 | 13,443.14 | 771.69 | 12,671.45 | 13,051.59 | | 20 | 13,051.59 | 13,443.14 | 391.55 | 13,051.59 | 0.00 | | Sum |  | 268,862.83 | 68,862.83 | 200,000.00 |  |   Consistency checks: The unpaid balance at maturity of the mortgage must be zero. The sum of all principal repayments must be equal to the borrowed amount. |

**iv) Payment calendar – synthetic approach**

the synthetic approach is based on the simulation of a sequence of prepaid mortgages

* take out the first-round mortgage for the period *T* and calculate the instalment
* pay the instalment and take out the second-round mortgage for the period *T* – 1 in an amount of the unpaid balance of the first-round mortgage and prepay this mortgage
* calculate the instalment for the second-round mortgage which will be equal to the instalment of the first-round mortgage,
* using the same approach in all subsequent periods results in the net cash flow which is identical to the cash flow of the original mortgage

|  |
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| PROOF |
| A synthetic approach in the compilation of the payment calendar exploits the following property of the level-payment mortgage,  This property can be verified by using formulas we derived earlier for the outstanding mortgage balance and the annuity factor. We have |

compilation of payment calendar

* calculate the *Annuity factor* in the first row using the respective formula
* calculate the *Regular instalment* in the first row as a product of the beginning balance and the annuity factor
* break up the instalment in the first row into an *Interest payment* (product of the beginning balance and the mortgage rate) and *Principal repayment* (residual value after deducting interest payment from the instalment)
* calculate the *Unpaid balance* in the first row by deducting the principal repayment from the beginning balance
* the unpaid balance in the first row becomes the beginning balance in the second row
* continue in the same way by filling all succeeding rows

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Time | Beginning  balance | Annuity  factor | Regular  instalment | Interest payment | Principal  repayment | Unpaid balance |
| 1 |  |  |  |  |  |  |
| 2 |  | A |  |  |  |  |
| . . . | . . . . . | . . . . . |  | . . . . . | . . . . . | . . . . . |
| T |  | A |  |  |  | 0 |

|  |
| --- |
| EXAMPLE |
| Make out the payment calendar for a mortgage with a principal of 200,000 USD, time to maturity of 20 years and an annual mortgage rate of 3%. Suppose that the mortgage is repaid in annual instalments.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | **Year** | **Beginning**  **balance** | **Annuity**  **factor** | **Regular**  **instalment** | **Interest**  **payment** | **Principal**  **repayment** | **Unpaid**  **balance** | | 1 | 200,000.00 | 0.067216 | 13,443.14 | 6,000.00 | 7,443.14 | 192,556.86 | | 2 | 192,556.86 | 0.069814 | 13,443.14 | 5,776.71 | 7,666.44 | 184,890.42 | | 3 | 184,890.42 | 0.072709 | 13,443.14 | 5,546.71 | 7,896.43 | 176,993.99 | | 4 | 176,993.99 | 0.075953 | 13,443.14 | 5,309.82 | 8,133.32 | 168,860.67 | | 5 | 168,860.67 | 0.079611 | 13,443.14 | 5,065.82 | 8,377.32 | 160,483.35 | | 6 | 160,483.35 | 0.083767 | 13,443.14 | 4,814.50 | 8,628.64 | 151,854.71 | | 7 | 151,854.71 | 0.088526 | 13,443.14 | 4,555.64 | 8,887.50 | 142,967.21 | | 8 | 142,967.21 | 0.094030 | 13,443.14 | 4,289.02 | 9,154.13 | 133,813.08 | | 9 | 133,813.08 | 0.100462 | 13,443.14 | 4,014.39 | 9,428.75 | 124,384.34 | | 10 | 124,384.34 | 0.108077 | 13,443,14 | 3,731.53 | 9,711.61 | 114,672.72 | | 11 | 114,672.72 | 0.117231 | 13,443.14 | 3,440.18 | 10,002.96 | 104,669.76 | | 12 | 104,669.76 | 0.128434 | 13,443.14 | 3,140.09 | 10,303.05 | 94,366.72 | | 13 | 94,366.72 | 0.142456 | 13,443.14 | 2,831.00 | 10,612.14 | 83,754.58 | | 14 | 83,754.58 | 0.160506 | 13,443.14 | 2,512.64 | 10,930.50 | 72,824.07 | | 15 | 72,824.07 | 0.184598 | 13,443.14 | 2,184.72 | 11,258.42 | 61,565.65 | | 16 | 61,565.65 | 0.218355 | 13,443.14 | 1,846.97 | 11,596.17 | 49,969.48 | | 17 | 49,969.48 | 0.269027 | 13,443.14 | 1,499.08 | 11,944.06 | 38,025.42 | | 18 | 38,025.42 | 0.353530 | 13,443.14 | 1,140.76 | 12,302.38 | 25,723.04 | | 19 | 25,723.04 | 0.522611 | 13,443.14 | 771.69 | 12,671.45 | 13,051.59 | | 20 | 13,051.59 | 1.030000 | 13,443.14 | 391.55 | 13,051.59 | 0.00 | | Sum |  |  | 268,862.83 | 68862.83 | 200,000.00 |  |   Consistency checks: The unpaid balance at maturity of the mortgage must be zero. The sum of all principal repayments must be equal to the borrowed amount. |

**5.2 Prepayments**

there are three main components of the mortgage cash flow

* interest payment
* scheduled principal repayment
* prepayments – payments in excess of regularly scheduled principal repayments

reasons for prepayments

* right of the mortgage borrower to pay off a part or all of the mortgage at any time (prepayment is an option given by the mortgage lender to the mortgage borrower)
* default on meeting mortgage obligation (the property is repossessed, sold and proceeds are used to pay off the mortgage)
* changes in the mortgage borrowers’s personal life, such as moving to another location, change of employment, purchase of a new house, divorce, etc.

**i) Prepayment formula**

… projected principal prepayments for the period *t*

… prepayment rate in the period *t* (based on the historical prepayment experience and on the current and expected economic conditions)

. mortgage balance at the beginning of the period *t* which includes all previous prepayments

… scheduled principal repayment for period *t*

relationship between the annual prepayment rate and the monthly prepayment rate

**ii) Prepayment risk**

prepayment risk means that prepayments occur at any unfavourable time from the point of view of the investor and that the timing and amount of prepayments are not known with certainty and are difficult to predict accurately

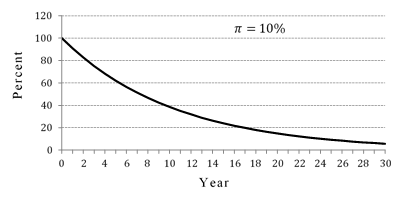
* contraction risk means that declining mortgage rates speed up prepayments so investors are forced to reinvest unplanned cash inflow at a lower market rate
* extension risk means that rising mortgage rates slow down prepayments so investors are deprived of cash inflow that could be invested at higher market rates

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| EXAMPLE |
| Make out the payment calendar for a mortgage with a principal of 200,000 USD, time to maturity of 20 years and an annual mortgage rate of 3%, assuming that the annual prepayment rate is 6 %. Suppose that the mortgage is repaid in annual instalments.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | **Year** | **Beginning**  **balance** | **Scheduled**  **instalment** | **Interest**  **payment** | **Principal**  **repayment** | **Prepay-**  **ments** | **Unpaid**  **balance** | | 1 | 200,000.00 | 13,443.14 | 6,000.00 | 7,443.14 | 11,553.41 | 181,003.45 | | 2 | 181,003.45 | 13,443.14 | 5,430.10 | 8,013.04 | 10,379.42 | 162,610.98 | | 3 | 162,610.98 | 13,443.14 | 4,878.33 | 8,564.81 | 9,242.77 | 144,803.40 | | 4 | 144,803.40 | 13,443.14 | 4,344.10 | 9,099.04 | 8,142.26 | 127,562.10 | | 5 | 127,562.10 | 13,443.14 | 3,826.86 | 9,616.28 | 7,076.75 | 110,869.07 | | 6 | 110,869.07 | 13,443.14 | 3,326.07 | 10,117.07 | 6,045.12 | 94,706.88 | | 7 | 94,706.88 | 13,443.14 | 2,841.21 | 10,601.94 | 5,046.30 | 79,058.65 | | 8 | 79,058.65 | 13,443.14 | 2,371.76 | 11,071.38 | 4,079.24 | 63,908.03 | | 9 | 63,908.03 | 13,443.14 | 1,917.24 | 11,525.90 | 3,142.93 | 49,239.20 | | 10 | 49,239.20 | 13,443.14 | 1,477.18 | 11,965.97 | 2,236.39 | 35,036.85 | | 11 | 35,036.85 | 13,443.14 | 1,051.11 | 12,392.04 | 1,358.69 | 21,286.12 | | 12 | 21,286.12 | 13,443.14 | 638.58 | 12,804.56 | 508.89 | 7,972.67 | | 13 | 7,972.67 | 8,211.85 | 239.18 | 7,972.67 | 0 | 0 | | 14 | 0 | 0 | 0 | 0 | 0 | 0 | | 15 | 0 | 0 | 0 | 0 | 0 | 0 | | 16 | 0 | 0 | 0 | 0 | 0 | 0 | | 17 | 0 | 0 | 0 | 0 | 0 | 0 | | 18 | 0 | 0 | 0 | 0 | 0 | 0 | | 19 | 0 | 0 | 0 | 0 | 0 | 0 | | 20 | 0 | 0 | 0 | 0 | 0 | 0 | | Sum |  | 169529,55 | 38341,72 | 131187,82 | 68812,18 |  |   Consistency check: The sum of all principal repayments and prepayments must be equal to the borrowed amount. |

**5.3 Inflation-linked mortgage**

**i) Tilt and affordability problems**

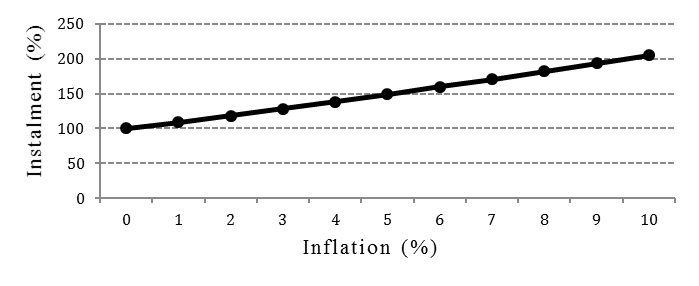
* tilt problem (erosion problem) arises when high inflation progressively erodes (tilts) the purchasing power of principal repayments so lenders are discouraged to provide adequate funding for the mortgage market



the graph shows that by the twentieth year an annual inflation of 10% reduces the purchasing power of the invested dollar to some 15% of its original value

from the investor’s point of view, the tilt effect can be addressed by including the expected inflation in mortgage rates

* affordability problem arises when high mortgage rates, including inflation, result in a high real value of instalments at the beginning of the mortgage’s life, hereby reducing the affordability of mortgages for large segments of the population



term of the mortgage: 20 years

real mortgage rate: 4 %

the graph indicates that for a 20-year level-payment mortgage with a 9% mortgage rate (composed of a real rate of 4% and inflation of 5%), the regular instalment would be about 50% higher than the instalment at zero inflation

an inflation of 10% would result in the instalment being more than twice as high as the instalment at zero inflation

**ii) Graduated-payment mortgage**

instalments of the graduated-payment mortgage grow at the rate of inflation leaving the real value of instalments unchanged

… regular instalment in the GPM plan

… base value for graduated instalments

… rate of inflation

pricing equation

the base value for graduated instalments is equal to the annuity that would be paid in case of zero inflation

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| --- |
| EXAMPLE |
| Compare the payment calendar of a 20-year level-payment mortgage with the payment calendar of a graduated-payment mortgage. Assume that the mortgage rate of the level-payment mortgage takes into account a real mortgage rate of 3% and a steady inflation of 8 %. The beginning balance of both mortgages is 200,000 USD.  Payment calendar of the level-payment mortgage:  mortgage rate =   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **Year** | **Beginning**  **balance** | **Regular**  **instalment** | **Interest**  **payment** | **Principal repayment** | **Unpaid**  **balance** | | 1 | 200,000.00 | 25,510.38 | 22,480.00 | 3,030.38 | 19,6969.62 | | 2 | 196,969.62 | 25,510.38 | 22,139.39 | 3,371.00 | 19,3598.62 | | 3 | 193,598.62 | 25,510.38 | 21,760.49 | 3,749.90 | 18,9848.72 | | 4 | 189,848.72 | 25,510.38 | 21,339.00 | 4,171.39 | 18,5677.34 | | 5 | 185,677.34 | 25,510.38 | 20,870.13 | 4,640.25 | 18,1037.09 | | 6 | 181,037.09 | 25,510.38 | 20,348.57 | 5,161.81 | 17,5875.28 | | 7 | 175,875.28 | 25,510.38 | 19,768.38 | 5,742.00 | 17,0133.28 | | 8 | 170,133.28 | 25,510.38 | 19,122.98 | 6,387.40 | 16,3745.88 | | 9 | 163,745.88 | 25,510.38 | 18,405.04 | 7,105.35 | 15,6640.53 | | 10 | 156,640.53 | 25,510.38 | 17,606.40 | 7,903.99 | 14,8736.54 | | 11 | 148,736.54 | 25,510.38 | 16,717.99 | 8,792.39 | 13,9944.15 | | 12 | 139,944.15 | 25,510.38 | 15,729.72 | 9,780.66 | 13,0163.49 | | 13 | 130,163.49 | 25,510.38 | 14,630.38 | 10,880.01 | 11,9283.48 | | 14 | 119,283.48 | 25,510.38 | 13,407.46 | 12,102.92 | 10,7180.57 | | 15 | 107,180.57 | 25,510.38 | 12,047.10 | 13,463.29 | 9,3717.28 | | 16 | 93,717.28 | 25,510.38 | 10,533.82 | 14,976.56 | 7,8740.72 | | 17 | 78,740.72 | 25,510.38 | 8,850.46 | 16,659.92 | 6,2080.80 | | 18 | 62,080.80 | 25,510.38 | 6,977.88 | 18,532.50 | 4,3548.30 | | 19 | 43,548.30 | 25,510.38 | 4,894.83 | 20,615.55 | 2,2932.74 | | 20 | 22,932.74 | 25,510.38 | 2,577.64 | 22,932.74 | 0.00 | | Sum |  | 510,207.64 | 310,207.64 | 200,000.00 |  |   Payment calendar of the graduated-payment mortgage:   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | **Year** | **Beginning**  **balance** | **Base**  **value** | **Price**  **index** | **Regular**  **instalment** | **Interest**  **payment** | **Principal repayment** | **Unpaid**  **balance** | | 1 | 200,000.00 | 13,443.14 | 1.0800 | 14,518.59 | 22,480.00 | -7,961.41 | 20,7961.41 | | 2 | 207,961.41 | 13,443.14 | 1.1664 | 15,680,08 | 23,374.86 | -7,694.78 | 21,5656.19 | | 3 | 215,656.19 | 13,443.14 | 1.2597 | 16,934.49 | 24,239.76 | -7,305.27 | 22,2961.46 | | 4 | 222,961.46 | 13,443.14 | 1.3605 | 18,289.25 | 25,060.87 | -6,771.62 | 22,9733.08 | | 5 | 229,733.08 | 13,443.14 | 1.4693 | 19,752.39 | 25,822.00 | -6,069.61 | 23,5802.69 | | 6 | 235,802.69 | 13,443.14 | 1.5869 | 21,332.58 | 26,504.22 | -5,171.65 | 24,0974.34 | | 7 | 240,974.34 | 13,443.14 | 1.7138 | 23,039.18 | 27,085.52 | -4,046.33 | 24,5020.67 | | 8 | 245,020.67 | 13,443.14 | 1.8509 | 24,882.32 | 27,540.32 | -2,658.01 | 24,7678.68 | | 9 | 247,678.68 | 13,443.14 | 1.9990 | 26,872.90 | 27,839.08 | -966.18 | 24,8644.86 | | 10 | 248,644.86 | 13,443.14 | 2.1589 | 29,022.73 | 27,947.68 | 1,075.05 | 24,7569.81 | | 11 | 247,569.81 | 13,443.14 | 2.3316 | 31,344.55 | 27,826.85 | 3,517.71 | 24,4052.10 | | 12 | 244,052.10 | 13,443.14 | 2.5182 | 33,852.12 | 27,431.46 | 6,420.66 | 23,7631.44 | | 13 | 237,631.44 | 13,443.14 | 2.7196 | 36,560.29 | 26,709.77 | 9,850.51 | 22,7780.93 | | 14 | 227,780.93 | 13,443.14 | 2.9372 | 39,485.11 | 25,602.58 | 13,882.53 | 21,3898.40 | | 15 | 213,898.40 | 13,443.14 | 3.1722 | 42,643.92 | 24,042.18 | 18,601.74 | 19,5296.66 | | 16 | 195,296.66 | 13,443.14 | 3.4259 | 46,055.43 | 21,951.34 | 24,104.09 | 17,1192.57 | | 17 | 171,192.57 | 13,443.14 | 3.7000 | 49,,739.87 | 19,242.05 | 30,497.82 | 14,0694.75 | | 18 | 140,694.75 | 13,443.14 | 3.9960 | 53719.06 | 15,814.09 | 37,904.97 | 10,2789.79 | | 19 | 102,789.79 | 13,443.14 | 4.3157 | 58,016.58 | 11,553.57 | 46,463.01 | 5,6326.78 | | 20 | 56,326.78 | 13,443.14 | 4.6610 | 62,657.91 | 6,331.13 | 56,326.78 | 0.00 | | Sum |  |  |  | 664,399.33 | 0.00 | 200,000.00 |  | |

comparison of level-payment and graduated-payment mortgages

* a GPM has smaller instalments at the beginning but high instalments at the end of the mortgage life that might be a problem for households whose income cannot keep pace with inflation
* negative amortization means that instalments are insufficient to cover the entire interest payment so the unpaid mortgage balance is increased by unpaid interest

negative amortization might be a problem if the growing unpaid balance exceeds the market value of the property so mortgage borrowers have incentive to default on their obligations

**iii) Other mortgage designs**

* adjustable rate mortgage (ARM) is a contract in which the mortgage rate is reset periodically (after one month, half a year, one year, two years etc.) in accordance with some appropriately chosen benchmark index (market-determined short-term interest rate, cost of funding of mortgage originators etc.)

caps in ARM are limits to a change in the contract at the reset date (protection of borrowers against sharp unfavourable changes in payment conditions)

periodic rate cap limits the change in the mortgage rate at the reset date

periodic payment cap limits the change in the regular instalment at the reset date

lifetime cap is the upper limit on the contract rate that could be charged over the life of the loan

a cap is an option that the lender has effectively sold to the homeowner (some of the expected loss for mortgage originators or investors caused by caps may be recouped by mortgage rates or other less favourable terms)

floors in ARM are lower limits on changes in the mortgage rate or payment amount at reset dates

floors can have a periodic as well as a lifetime nature

* hybrid mortgage is a contract that shares features of fixed-rate and adjustable-rate mortgages

a convertible adjustable rate mortgage can be transformed into a fixed-rate mortgage if rates fall below some predetermined level

a convertible (reducible) fixed rate mortgage can have the mortgage rate lowered

the borrower has the choice of converting and the lender charges a nominal fee for conversion

**5.4 Foreign-currency mortgage**

foreign-currency mortgage is provided and repaid in a currency which is different from the currency of the country in which the borrower is resident

mortgage payments may be made in domestic currency, but their size changes in line with changes in the exchange rate of the foreign currency

the interest rate charged is based on the foreign interest rates relevant to the currency in which the mortgage is denominated

incentives

* significantly lower interest rates on borrowing in foreign currency than interest rates on borrowing in domestic currency
* prospects for the strengthening of the domestic currency against foreign currency (paid instalments denominated in foreign currency become smaller in terms of domestic currency)

risks

* open currency position (currency mismatch) – revenues in one currency are used for paying outlays in another currency
* mortgage borrower is squeezed by the weakening of the domestic currency as instalments become larger in terms of domestic currency

**5.5 Securitization with mortgages**

securitization comprises various techniques of financial engineering in which a variety of assets (mortgages, credit card receivables, bank loan and others) are pooled and used as a source of cash flow (collateral) for the creation of new securities called asset-backed securities(ABS)

securitization with mortgages gives rise to mortgage-backed securities (MBS) or collateralized mortgage obligations (CMO)

**i) participants in securitization**

**Structurer (conduit)**

**Investors**

**Originator**

MBS 1

Mortgage 1

Investor 1

Mortgage 1

MBS

MBS 2

Mortgage 2

Pool

Mortgage 2

MBS 3

Mortgage 3

Mortgage 3

Price

Price

MBS Y

Investor Z

Mortgage X

Mortgage X

Guarantees

Fee

**Sponsor (insurer)**

**Credit enhancement**

* originatoris a financial institution (mortgage bank, commercial bank, thrift and others) which provides mortgage loans

originator arranges mortgage loans in a pool (collection of mortgages) that is sold to a financial institution called the structurer

* structurer(conduit) is an entity that purchases a pool of mortgages and issues MBS whose cash flow is a transformation of the cash flow from the underlying pool

specific structures of derived cash flow give rise to specific types of MBS

* servicer is a financial institution (usually identical with the originator of the pool) which administers the mortgage loans in exchange for a fee (collects mortgage payments, forecloses property of delinquent debtors, transfers the income to MBS investors, etc.)
* sponsoris an agency (usually a government-owned or government-sponsored entity) which provides insurance or other forms of guarantees related to the cash flow from MBS with the aim to reduce credit risk for MBS investors

US institutions: Federal National Mortgage Association (Fannie Mae), Federal Home Loan Mortgage Corporation (Freddie Mac), Government National Mortgage Association (Ginnie Mae)

agency deals are backed to some extent by guarantees of government or government-sponsored institutions (timely payment of interest, principal or both of them)

a major risk concern in agency deals is prepayment risk (contraction risk or extension risk)

private-label deals are arranged by private financial institutions without any government guarantees

a major risk concern in non-agency deals is credit risk (mortgage borrowers will default by either refusing to pay or declaring bankruptcy)

credit enhancement is a generic name for different types of support that aim to absorb expected losses and provide some protection against unexpected losses

* excess spread is the difference between a higher average interest rate paid by the mortgage pool and a lower average interest rate paid to the MBS holders
* over-collateralization is the difference between a higher market value of the mortgage pool and a lower market value of the MBS issued to investors
* third-party guarantees are provided by private financial institutions
* diversification means the application of various limits (per borrower, size of loans, industry, geographical region, etc.) in order to avoid concentration of credit risk in a particular economic segments

the amount and type of credit enhancement is evaluated by rating agencies given the target rating for securities included in the securitization structure

**ii) Some securitization techniques**

* passthrough securities are MBS which redirect the cash flow from the underlying pool of mortgages on a pro rata basis to all MBS holders

by holding passthroughs, the investor holds a diversified portfolio of mortgages, in this way reducing substantially unsystematic risk and leaving only systemic risk

liquidity of passthroughs is considerably better than that of individual mortgages (there is no need to dispose of loans one by one)

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| EXAMPLE |
| A structurer formed a pool of 500 mortgages, each of them in an amount of 100,000 USD. Against this pool the structurer issued 5,000 passthroughs, each of them in a nominal amount of 10,000 USD. Now, there are two possibilities of how to invest one million USD.   1. Direct purchase of 10 individual mortgages (1,000,000:100,000 = 10). In this case, a default of one mortgage represents a loss of 100,000 USD for the investor, which is 10% of the invested amount (100,000:1,000,000 = 0,1 = 10%). 2. Purchase of 100 passthroughs (1,000,000:10,000 = 100). In this case, a loss of 100,000 USD from the default of one mortgage in the pool is distributed evenly among 5,000 passthroughs, which is 20 USD per passthrough. The holder of 100 passthroughs thus incurs a loss of 2,000 USD, which represents only 0.2% of the invested amount (2,000:1,000,000 = 0.002 = 0.2%). |

* stripped mortgage-backed securities (strips) are created by altering the distribution of principal and interest from a pro rata distribution to an unequal distribution

IO class (interest only) receives all of the interest from an underlying pool

PO class (principal only) receives entire principal from an underlying pool

properties of IOs and POs

* mortgage rates fall ⇨ prepayments speed up

prices of PO increase (PO holders recover principal repayments sooner and this cash inflow is discounted at a lower discount rate)

prices of IO decline (unpaid principal declines faster, thereby lowering the base for calculating the interest; this effect is usually stronger that the offsetting effect of discounting at a lower discount rate)

* mortgage rates rise ⇨ prepayments slow down

prices of PO decline (it takes longer to recover principal repayments and this cash inflow is discounted at a higher discount rate)

prices of IO increase (unpaid principal declines more slowly which improves the base for calculating the interest; this effect is usually stronger that the offsetting effect of discounting at a higher discount rate)

* sequential structure creates a set of bond classes (tranches) with priority rules for distributing the cash flow from the mortgage pool among holders of individual classes

principal (both scheduled and prepaid) is directed first to bond class A until it is completely paid off, then all principal payments are made to bond class B until it is completely paid off and so on

interest is distributed to each class on the basis of the outstanding principal

in comparison to the underlying pool of mortgages, bond classes in the sequential structure differ in terms of their maturities and their exposure to prepayment risk which better reflects the preferences of different types of investors

* subordination structurecreates classes with priority rules for absorbing losses from the underlying pool of mortgages

hierarchy of classes: equity class, junior class, mezzanine class, senior class

the equity class picks the losses first until it is completely wiped out, then the junior class absorbs the losses until it is completely wiped out and so on

the higher the class in the subordination hierarchy, the better its credit rating and the lower its yield

**iii) Pros and cons of securitization**

advantages

* reduction of funding costs (securitization means secured lending, desired credit rating assigned to MBS can be achieved by appropriate credit enhancements)
* managing corporate risk (credit and interest rate risks are removed by selling the pool of risky mortgages)
* reduction of regulatory capital (lower retained risk with the originator results in lower requirements for regulatory capital)

critiques

* lax underwriting standards (originate-and-distribute practices reduce motivation to properly asses the credit quality of borrowers)
* greater opaqueness of risk exposures (some securitization structures became too complicated to be understood by investors; high complexity leads to over-reliance on the ratings assigned by rating agencies)
* flawed incentives (rating agencies are paid primarily by those who are the issuers of MBS rather than by those who are the investors in MBS)
* reduced effectiveness of monetary policy (securitization allows borrowers direct access to end lenders, thus reducing the traditional role of banks in the financial intermediation)

**6 MONEY MARKET SECURITIES**

money market securities is an umbrella term for all short-term financial instruments (predominantly with maturities of less than a year) which are based on interest (actually paid or implied by pricing)

capital market securities encompass interest-based securities with maturities in excess of one year (bonds, mortgages and others)

fixed-income securities are all instruments (belonging both to money and capital markets) that provide a return in the form of fixed periodic payments (unlike variable-income securities)

main money market instruments

* classical borrowing/lending arrangements: time deposit, certificate of deposit (CD), Treasury bill (T-bill), commercial paper, bill of exchange
* more complex arrangements: sale and repurchase agreements (repo), forward rate agreements (FRA), short-term interest rate futures

**6.1 Money market conventions**

**i) Annual versus periodic interest rate**

annual interest rate is a rate quoted on an annual basis (it assumes that the maturity of the quoted instruments is one year)

the size of the annual interest rate depends on the convention of how many days make up one year

* ACT/365 convention assumes 365 days in a year (even for leap years)
* ACT/360 convention assumes 360 days in a year

conversion between conventions

periodic interest rate is an annual rate adjusted to an exact number of days in the period in question

the size of the periodic interest rate depends on the type of interest

* simple interest

periodic IR = × annual IR

* compound interest

periodic IR =

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| EXAMPLE |
| A 20-day time deposit promises to pay 10% on an ACT/365 basis.  This is the annual yield quoted on an ACT/360 basis:  This is the true yield generated over the 20-day interest period based on ACT/365 conventions:  This is true yield generated over the 20-day interest period based on ACT/360 conventions:  We see that the actual yield over a given period does not depend on the conventions in which the annual yield is quoted. |

**ii) Simple return versus effective yield**

simple return (SR) reproduces the yield of a money market instrument on the basis of simple interest

effective yield (EY) reproduces the yield of a money market instrument on the basis of compound interest

**iii) Quotation on a yield versus a discount basis**

quotation on a yield basis assumes that the instrument is issued for its nominal value and redeemed at its nominal value plus periodic interest

*r* … annual interest rate

quotation on a discount basis assumes that the instrument is issued with a discount from its nominal value and redeemed at its nominal value

*d* … annual discount rate

discount rate is always lower than the corresponding simple return

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| EXAMPLE |
| What is the simple return of a one-month money market instrument which is selling with an annual discount of 10%?  At what discount should a one-month money market instrument be sold to generate an annual yield of 10%?  What is the effective yield of a one-month money market instrument which is quoted with a simple return of 10%? |

**iv) Money market yield curve**

money market yield curve shows how short-term money market interest rates vary with term to maturity (time unit is one month)

* offer rates are money market interest rates at which banks are willing to lend funds to other banks

LIBOR (London Interbank Offer Rate) is an acronym for offer rates quoted for a number of different maturities (ranging from overnight to one year) and major currencies in the London interbank market

yields on many short-term instruments are set with the reference to LIBOR rates

similar reference rates are used in other money markets centres: EURIBOR (Eurozone), PRIBOR (Prague), NYBOR (New York), TIBOR (Tokyo), etc.

* bid rates are money market interest rates at which banks are willing to borrow funds from other banks

LIBID, PRIBID, NYBID, TIBID are money market borrowing rates in corresponding money market centres

interpolation of odd-date yields (yields for dates that do not coincide with generally quoted dates expressed in a number of months)

proposition about similar triangles

money market implied forward rates are forward rates that are consistent with an observed short-term yield curve, namely with the interest rate parity condition

a horizontal yield curve implies slightly falling instead of unchanging forward rates

this effect is not very strong and is suppressed when using the approximation formula of forward rates

**6.2 Certificate of deposit**

certificate of deposit (CD) is a negotiable security issued by banks to raise liquidity

CD typically pays one coupon at maturity which is not usually longer than one year

CD with longer maturities may pay more than one coupon in periodic intervals; the holder of this CD is exposed to reinvestment risk (similar to coupon bonds)

**i) Holding-period yield**

HPY is a simple return earned in the period between the time of purchasing and the time of selling the CD

M-maturity

P-purchase

I-issuance (

S-sale

… PM-day market yield at time of purchasing CD

… SM-day market yield at time of selling CD

… number of days between indicated points of time *I*, *P*, *S* and *M*

the CD purchasing price is equal to the amount obtained at maturity, discounted to the point of purchasing the CD at a -day discount rate

the selling price is equal to the amount obtained at maturity, discounted to the point of selling the CD at a -day discount rate

formula for the holding-period yield

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| EXAMPLE |
| A 90-day CD with a 6% coupon was purchased 24 days from its issuance. At that time the yield curve was flat at a level of 5.5%. The CD was held for 30 days and then sold when the yield fell to 5.0%. What is the holding-period yield? Use the ACT/360 convention. |

**6.3 Treasury bill**

Treasury bill (T-bill) is a short-term security issued by ministries of finance with the aim to raise short-term financing

T-bills are quoted on a discount basis (security pays no explicit interest and remuneration is in the form of a discount to its par value)

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| EXAMPLE |
| A three-month Treasury bill with a face value of 1,000 USD was issued with an annual discount of 10%. What is its issuing price? What is the corresponding simple return? Use the ACT/360 convention. |

**i) Bond-equivalent yield**

long-term bonds maturing in less than one year become direct investment alternative to short-term government bonds so the yields of both instruments must be comparable regardless of different conventions used in capital and money markets

bond-equivalent yield (BEY) expresses the yield of a T-bill as if it were a bond which has less than one year to maturity

BEY is defined as a coupon rate of a hypothetical par bond, which was issued at a nominal value equal to the current price of the T-bill and which has a maturity equal to the current maturity of the T-bill (par bonds are characterized by equality between the coupon rate and the yield to maturity)

* a bond has less than six months to maturity ⇒ an equivalent bond pays only one coupon at maturity

calculation of BEY (by definition equal to *c*) is based on the solution of the equation

left side: cash flow of the T-bill received at maturity

right side: cash flow of the equivalent bond received at maturity whose components are

1. – nominal value of the equivalent bond (by definition equal to the current price of the T-bill)
2. – coupon of the equivalent bond, adjusted to the length of the coupon period of the equivalent bond (by definition equal to the time to maturity of the T-bill)

* a bond has more than six months to maturity ⇒ equivalent bond pays two coupons, the first one six months before maturity and the second one at maturity

calculation of BEY, by definition equal to *c*, is based on the solution of the quadratic equation

left side: cash flow of the T-bill received at maturity

right side: cash flow of the equivalent bond received at maturity whose components are

1. – penultimate coupon of the equivalent bond, adjusted to the length of the coupon period (by definition equal to the time to maturity minus half of the year)
2. – reinvestment of penultimate coupon for the remaining half of the year (product of contents of both parentheses is the accumulated value of the reinvested penultimate coupon)
3. – nominal value of the equivalent bond received at maturity (by definition equal to the current price of the T-bill)
4. – last coupon of the equivalent bond received at maturity, adjusted to the length of the coupon period (by definition equal to half of the year)

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| EXAMPLE |
| A Treasury bill with a current price of 90 USD and a face value of 100 USD has 250 days to maturity. Calculate its bond-equivalent yield. Use the ACT/365 convention.  The bill has more than half the year to maturity so the equivalent bond will pay two coupons. The total cash flow of the equivalent bond at its maturity will thus be composed of the following components:  the value of the first coupon =  the interest earned by reinvesting the first coupon =  the value of the principal =  the final coupon =  The bond-equivalent yield is obtained by solving the quadratic equation  .  We have |

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| MATH NOTE |
| Solving a single-variable quadratic equation is an exercise of elementary algebra. Just to recapitulate, this equation is any equation having the form  where *a*, *b* and *c* are known numbers. The equation can be solved by using the quadratic formula  The equation for a bond-equivalent yield can be rearranged in line with the general form of quadratic equations. We get |

**6.4 Sale and repurchase agreement**

sale and repurchase agreement (repo)is short for a deal whereby two transactions, sale of a security and a reversal of the first transaction, are dealt as a package in one agreement

Bonds

Repo seller

(lender)

Bonds

Cash

Cash + interest

Repo buyer

(borrower)

repo terminology is used in reference to the bond market, not the cash market

* the seller of the repo is the party who sells bonds and repurchases them later whereas the buyer is the party who buys bonds and resells them later
* the lender in the repo is the party who lends the bonds over the repo term whereas the borrower is the party who borrows bonds over the repo term

reverse repo is the same arrangement viewed from the other party’s side (initial purchase of security followed by subsequent sale) so the repo to one party is the reverse repo to the other party

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| EXAMPLE |
| A dealer sold a one-month repo at a repo rate of 3.5%. As collateral, the repo used government bonds in a nominal amount of 200,000,000 EUR. The bond paid an annual coupon of 10% and its market price at the start of repo was 1,150 per 1,000 principal.  Transactions at the start of the repo deal:   1. The dealer sold bonds in the nominal amount of 200,000,000 EUR. 2. The dealer obtained the cash equal to the market price of bonds, which is   Transactions at maturity of the repo deal:   1. The dealer repurchased bonds in the nominal amount of 200,000,000 EUR. 2. The dealer paid the amount of cash |

**i) Legal versus economic treatment of repo**

* legal treatment (seller versus buyer)

ownership of the bond passes from the bond’s seller to the bond’s buyer for the period of the repo

if the bond’s seller (cash borrower) defaults on the cash payment in the reversal deal, then the bond’s buyer (cash lender) can keep the bond (the repoed security is excluded from bankruptcy proceedings)

repo provides dual protection against credit risk (credit standing of the counterparty and credit quality of the collateral)

* economic treatment (lender versus borrower)

repo is a type of secured lending in which the cash lender (bond’s borrower) holds the bond as collateral

haircut (margin) is a percentage by which the collateral value exceeds the cash loan (the amount of haircut depends on the relative creditworthiness of the two parties)

borrowed cash = market value of bonds × (1 – haircut)

haircut > 0 ⇒ borrowed cash < value of bonds = over-collateralized repo

haircut < 0 ⇒ borrowed cash > value of bonds = under-collateralized repo

cash lender is rewarded by being paid interest (repo rate)

cash lender (bond buyer) is not entitled to a coupon that is paid by the underlying bond despite the fact that he/she is officially the owner of the bond (his/her reward consists of the interest based on an agreed repo rate)

the entire coupon obtained for a repo term is therefore transferred to the original owner of the bond

**ii) Other notions in repo transaction**

* cash-driven versus security-driven repo

in a cash-driven repo, the deal is based on a particular amount of cash to be borrowed and the exact nature of the collateral is not important

in a security-driven repo, the deal is initiated by the need to borrow a particular amount of security which is called the special (the more special the security, the lower the repo rate)

* marking to market

the cash lender continually recalculates the value of the collateral in order to ensure that it is of adequate value

if the value of the collateral has fallen, the cash lender may take a margin call requiring the cash borrower to transfer more collateral (in terms of more securities or in cash)

if the value of the collateral rises, the cash borrower can make a margin call requiring the cash lender to return some of the collateral

a variation margin is the amount of collateral transferred between the seller and the buyer in response to a margin call

* substitution

repo documentation may include the right of the security’s seller to change the security used as collateral during the period of the repo

* maturity

repo period has a length from one day (overnight repo) to several months

the term repo means that the repo period is fixed and agreed upon in advance

the open repo means that either party may terminate the repo at any time (requiring some notice), otherwise the repo is rolled over each day

* custody repo

in a custody repo another organisation acts as custodian for the collateral (a custodian holds the security in a separate account on the buyer’s behalf)

the custodian performs several duties (marking to market, daily reports, substitutions and others)

the buyer cannot use the security in other repo deals

* bid-ask spread

by using a bid-ask spread, the dealer announces his price offer to purchase a given asset (bid price) and his price offer to sell the same asset (ask price)

in traditional borrowing and lending, the bid price indicates at what interest rate the dealer is willing to buy a deposit (borrow money), and the ask price indicates at what interest rate the dealer is willing to sell a deposit (lend money) ⇒ the bid price will always be lower than the ask price

in the repo market, the bid price indicates at which repo rate the dealer is willing to buy a bond (lend money), and the ask price indicates at which repo rate the dealer is willing to sell a bond (borrow money) ⇒ the bid price will always be higher than the ask price

**iii) Financing long position in bond market**

a trader who wants to buy a bond on the bond market simultaneously sells the bond in a repo in order to obtain the funds needed to pay for the bond purchased in the bond market (on the bond market the trader acts as the buyer of the bond and on the repo market the trader acts as the seller of the bond)

the feasibility of the deal is ensured by the settlement process which takes place at the end of the trading day when all payments are netted across all transactions that transpired during the day (it does not matter in what order the transactions were concluded)

Dealer

Repo deal

Purchasing bond

Paying cash

Using bond as collateral

Borrowing cash

Bond market

leverage – to buy the bond, the investor needs to put in only the haircut amount

|  |
| --- |
| EXAMPLE |
| A speculator expected an appreciation of a bond price. He purchased the bond using a two-week repo at a repo rate of 4%. The borrowed amount in the repo deal was subject to a haircut of 2%. At maturity of the repo, the bond price was higher by 5%. What is the annual simple return of this speculative strategy? And what would be a negative return assuming that at maturity the bond price was lower by 5%?  The bond price went up:  The bond price went down:  For comparison, the yield and loss in the case of a traditional speculation, which consists of buying and selling the bond without a leverage, would be much lower, namely |

**iv) Covering short position in bond market**

a trader who wants to sell a bond on the bond market, simultaneously buys the bond in a reverse repo deal and uses the cash received from the sale of the bond on the bond market to pay for the bond in the repo market (on the bond market the trader acts as the seller of the bond and on the repo market the trader acts as the buyer of the bond)

the feasibility of the deal is ensured by the settlement process which takes place at the end of the trading day when all payments are netted across all transactions agreed during the day (it does not matter in what order the transactions were concluded)

Dealer

Reverse repo deal

Selling bond

Receiving cash

Borrowing bond

Lending cash

Bond market

**v) leveraging bond portfolio**

a portfolio manager uses bonds as collateral in a series of repo transactions in order to borrow cash with the aim to buy even more bonds

leveraging can be repeated as often as prudence and margin requirements allow

Total exposure = 100

Step 1

Investment 100

Cash 100

Bonds 100

Bond market

Step 2

Cash 98

Cash 98

Bonds 100

Repo market

Bonds 98

Bond market

Total exposure = 198

Step 3

Cash 96

Cash 96

Bonds 98

Bonds 96

Bond market

Repo market

Total exposure = 294

maximum exposure on the bond market

a 2% haircut may result in a 50-fold increase in the speculative position

decline in bond prices will trigger margin calls for delivering more collateral in the whole chain of repo transactions ⇒ costly liquidation of repo position by selling bonds at depressed prices

**vi) Other uses of repo**

* reducing cost of borrowing

a corporation can borrow more cheaply if it has a bond in its investment portfolio which is in short supply in the market

such a bond can be ‘repoed’ (offered for sale in a repo transaction), which means to borrow funds at the repo rate

the attractiveness of the bond would be reflected in a favourable repo rate, which could be lower than the interest rate in conventional borrowing

* delivery process in interest rate futures contract

the terminology which is used in repo operations can be also seen in the physical delivery of long-term interest rate futures contracts (since futures trading is dealt with in other parts of this course, the following remarks limit themselves to only basic facts)

selling party in a futures contract (called the short) arranges the cash-and-carry transaction

the trader borrows an appropriate amount of cash which is used for purchasing a bond

this bond is held until the maturity of the futures contract

the proceeds from the futures sale are used for repaying the debt together with interest

this transaction can be viewed as a special case of reverse repo

it starts with the purchase of the bond in the capital market (cash is lent against collateral from the perspective of a repo)

it ends with the delivery of the bond into the futures contract (borrowed cash is returned from the perspective of the repo)

the size of the lent and paid back cash amounts determines the rate at which this hypothetical loan was extended; such a rate is called an implied repo rate

the cheapest-to-delivery bond is the bond that generates the highest hypothetical lending rate or the highest implied repo rate in the cash-and-carry transaction

* major tool of monetary policy

in injecting repo, central banks supply liquidity to the banking sector by purchasing eligible securities for the term of the repo transaction (they are doing reverse repos as a form of secured lending of funds)

in withdrawing repo, central banks withdraw liquidity from the banking sector by selling securities for the term of the repo

prevailing repo terms are over-night, one week, and two weeks