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| Swaps and credit derivatives  Lecture notes  O.D. Lecturing Legacy |

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**1 INTEREST RATE SWAP**

**1.1** **The swap mechanism**

interest rate swap is a financial contract that commits two counterparties to exchange two streams of interest payments over an agreed period, each of them calculated using a different interest rate

* interest payments are applied to a common notional principal amount; there is no exchange of principal, this figure is used only for the purpose of calculating the interest payments exchanged
* interest payments are netted, reducing the counterparty’s credit risk
* interest rate swaps are derivative instruments because they derive their cash flows from underlying cash instruments without the need to own these instruments for funding the payments
* interest rate swaps are off-balance sheet instruments because they do not appear on an investor’s balance sheet and affect only profit and loss accounts
* *termination of a swap* is the cancellation of the swap contract in which case one party compensates the other party for the loss of the expected profit over the remainder of the swap’s life
* *assignment of a swap* is the sale of the swap by one of the parties to a third party by an agreement of the other party

**1.2** **Coupon swap**

coupon swap is a fixed-against-floating interest rate swap that involves the exchange of an interest stream based on a fixed interest rate (fixed leg of the swap) for an interest stream based on a floating interest rate (floating rate of the swap)

* a fixed interest rate is unchanged over the life of the swap so the resulting stream of interest payments consists of equal amounts, each calculated at a rate known in advance
* a floating interest rate applies to an individual interest period and is reset at the beginning of the next interest period

**i) Buyer/seller convention**

* the buyer of a coupon swap is the party who pays the fixed rate and receives the floating rate
* the seller of a coupon swap is the party who receives the fixed rate and pays the floating rate

Swap buyer

Fixed interest

Swap seller

(seller)

Floating interest

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| EXAMPLE |
| Assume a five-year US dollar coupon swap involves the exchange of a fixed rate of 7.5 % paid semi-annually for a three-month LIBOR. Interest payments are based on a notional amount of 200 mil USD.  There will be 10 fixed interest payments paid at the end of each half-year interest period:  fixed interest payment =  There will be 20 different floating interest payments paid at the end of each quarter:  floating interest payment =  where denotes the value of the three-month LIBOR at the beginning of the *t*-th interest period. |

**ii) Quotation conventions**

|  |  |  |  |
| --- | --- | --- | --- |
| Maturity | Bid-ask rates  (two-way prices) | Spread over benchmark | Swap rate |
| 1 | 5.70/5.75 | 20/25 | 5.725 |
| 2 | 5.83/5.91 | 23/31 | 5.870 |
| 3 | 6.01/6.11 | 25/35 | 6.060 |

bid rate is the interest rate at which the dealer is prepared to buy the swap; it is the rate the dealer is willing to pay in the swap’s fixed leg

ask rate is the interest rate at which the dealer is prepared to sell the swap; it is the rate the dealer is willing to accept in the swap’s fixed leg

dealer’s spread is the difference between the two rates; it is the source of dealer’s profit earned from arranging a matching swap pair

swap rate is the average of bid and ask interest rates

benchmark interest rate is usually the yield of the most liquid government bond whose maturity coincides with the term of the swap

**iii) Other types of interest rate swaps**

basis swap is a floating-against-floating interest rate swap that involves a variety of combinations of floating interest rates

* different tenors of the same interest rate (i.e. a three-month LIBOR against a six-month LIBOR)
* the same or different tenors of different interest rates (i.e. a three-month LIBOR against a three-month Treasury)
* the same tenor of the same interest rate but with one carrying a margin (i.e. 3-month LIBOR against 3-mnth LIBOR plus 50 basis points)

the convention to call the parties of the basis swap the buyer and the seller is ambiguous; each party should be described in terms of the interest stream payed and received

generic swap (straight swap, plain vanilla swap) denotes the simplest construction of a swap contract (constant notional amount, unchanging fixed rate, flat floating rate with no margins, regular interest payments, immediate start, no special risk features)

esoteric (exotic) swaps are swaps that differ in one or more features that are characteristic of generic swaps

**1.3** **Speculative strategies using interest rate swaps**

classification of speculation strategies

* risk taking strategies – bets on the direction of future interest rate movements
* risk transforming strategies – changes in the type of interest rate risk exposure

**i) Risk taking strategies**

* using a coupon swap

Swap buyer

Fixed interest

Swap seller

(seller)

Floating interest

the buyer of a coupon swap expects the interest rate differential to change in such a way that the floating interest rate will rise above the fixed rate

the seller of a coupon swap must have the opposite expectation that the floating interest rate will fall below the fixed rate

analogy with asset-liability mismatching (*gapping*)

instead of buying the swap the speculator issues a bond that pays a fixed coupon and rolls over a short term deposit that generates floating interest

a swap represents a cheaper alternative to gapping because it achieves the same effects without the need to carry out transactions with cash instruments (buying the bonds, opening the term deposit)

* using a basis swap

Trader A

6-month Libor

Trader B

(seller)

3-month Libor

counterparties in a basis swap expect that the differential between the two short-term interest rates will change in such a way that the net effect will increase the overall profitability of the swap

**ii) Risk transforming strategies**

* transformation of a fixed-interest liability to a floating-interest liability

Issuer

Fixed coupon

Dealer

(seller)

Swap

an issuer issued a fixed coupon bond and wants to benefit from an expected fall in interest rates

by selling a coupon swap, the issuer will effectively pay a floating interest while still having the bond on his/her balance sheet

this strategy creates a synthetic floating-interest liability

* transformation of a floating-interest liability to a fixed-interest liability

Issuer

Variable coupon

Dealer

(seller)

Swap

an issuer issued a floating rate bond and wants to benefit from an expected rise in interest rates

by buying a coupon swap, the issuer will effectively be paying a fixed interest regardless of fluctuations of short-term interest rates while still having the bond on his/her balance sheet

this strategy creates a synthetic floating-interest liability

* transformation of a floating-interest asset to a fixed-interest asset

Investor

Variable coupon

Dealer

(seller)

Swap

an investor purchased a floating rate bond and wants to benefit from an expected fall in interest rates

by selling a coupon swap, the investor will effectively be receiving a fixed interest regardless of fluctuations of short-term interest rates while still having the bond in his/her investment portfolio

the strategy creates a synthetic fixed-interest asset

* transformation of a fixed-interest asset to a floating-interest asset

Investor

Variable coupon

Dealer

(seller)

Swap

an investor purchased a fixed coupon rate bond and wants to benefit from an expected increase in interest rates

by buying a coupon swap, the investor will effectively be receiving a floating interest while still having the bond in his/her investment portfolio

this strategy creates a synthetic fixed-interest asset

**iii) Transformation of a hedged position to a speculative position**

* speculation on an interest rate rise

Bank

Dealer

floating rate

floating rate

fixed rate

floating rate

a bank initially has no exposure to interest rate risk because both assets and liabilities derive their cash flows from floating interest rates

if the bank wishes to take a risk position based on an anticipated interest rate increase, it could do so by buying a coupon swap

the bank creates a synthetic interest rate mismatch that benefits from the fall of short-term interest rates

* speculation on an interest rate fall

Bank

Dealer

floating rate

floating rate

fixed rate

floating rate

a bank initially has no exposure to interest rate risk because both assets and liabilities derive their cash flows from floating interest rates

if the bank wishes to take a risk position based on an anticipated interest rate decrease, it could do so by selling a coupon swap

the bank creates a synthetic interest rate mismatch that benefits from the rise of short-term interest rates

analogous diagrams can be drawn for a situation in which both assets and liabilities derive their cash flows from fixed interest rates

**1.4** **Hedging strategies using interest rate swaps**

a hedge is a risk taken with the aim to offset an opposite risk, thereby creating a risk free position

swaps can provide the necessary offsetting risk

**i) Hedging with coupon swaps**

* a hedge against an interest rate rise

Bank

Dealer

fixed rate

floating rate

fixed rate

floating rate

a bank is exposed to the risk that interest rates will rise because its variable cost of funding may increase without an offsetting increase in the fixed returns on its assets

by buying a coupon swap, an increase in the floating rate paid will be offset by an increase in the floating rate received in the swap

this hedging scheme can be applied by mortgage lenders that fund long-term fixed rate mortgages from short-term floating rate deposits

* a hedge against an interest rate fall

Bank

Dealer

floating rate

fixed rate

fixed rate

floating rate

a bank is exposed to the risk that interest rates will fall because its variable returns on its assets may decrease without an offsetting decrease in the variable cost of funding

by selling a coupon swap, a fall in the floating rate received will be accompanied by an offsetting fall in the floating rate paid in the swap

this hedging scheme can be applied by mortgage lenders who fund floating rate mortgages from fixed rate liabilities such as bonds and longer-term certificate of deposits

**ii) Hedging with basis swaps**

Bank

Dealer

3-year interest

6-monthLIBOR

3-monthLIBOR

3-year interest

3-month LIBOR

6-month LIBOR

Customer

Customer

a bank concluded two opposite coupon swaps with different customers and is left with a basis risk between a three-month and a six-month LIBOR

a basis swap can hedge the risk of an unparalleled shift in short-term interest rates

**1.5** **Warehousing of coupon swaps**

warehousing comprises various techniques of hedging interest rate risk, which stems from concluding matching coupon swaps in different moments of time

*fully hedged position*

Dealer

fixed rate

fixed rate + spread

floating rate

floating rate

Customer

Customer

interest rate risks the dealer is exposed to during the search for a matching swap

* the dealer is the payer of a fixed rate in the earlier swap ⇨ an interest rate fall may result in a higher fixed rate paid in the earlier swap than the fixed rate received in the later swap
* the dealer is the recipient of a fixed rate in the earlier swap ⇨ an interest rate rise may result in a lower fixed rate received in the earlier swap than the fixed rate paid in the later swap
* changes in short-term interest rates during the searching period are not such a problem because the interest rate discrepancy will last only until the next reset date of the floating rate

**i) Warehousing with bonds**

* dealer is the payer of a fixed rate in the earlier swap and is concerned that long-term interest rates may fall during the searching period

Dealer

Money

market

5-year interest

Cash

Overnight rate

5-year bond

6-month LIBOR

Cash

Earlier customer

Bond

market

the dealer arranges a rolled-over overnight borrowing whose proceeds are used to buy bonds with the same maturity as the swap

the cost of borrowing is financed by the floating interest received in the earlier swap; the dealer is exposed to some basis risk because overnight rates are repriced each day

if the five-year interest rate falls, a higher selling price of bonds offsets a permanent income loss on the fixed legs of the matching swaps

the purchase of bonds could be alternatively financed using a repo instead of direct borrowing in the money market

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| EXAMPLE |
| A swap dealer arranged a three-year interest rate coupon swap with a notional amount of 100,000 USD, in which he/she pays a three-year interest rate of 10.5% once a year in exchange for receiving a six-month LIBOR. The dealer warehoused the swap by purchasing three-year bonds with a nominal value of 1,000 USD and a coupon of 10% paid annually. How effective was the hedge, assuming that the three-year interest rate dropped by 1.5 percentage points just after the warehousing strategy was established? The swap’s fixed rate of a corresponding maturity can be used as the discount rate.  The present value of the permanent loss from paying 10.5% in the earlier swap and receiving 9% in the later swap:  Purchasing price of the bond:  Selling price of the bond:  Number of bonds purchased in the warehousing operation:  Total capital gain:  The capital gain from the warehousing strategy lagged slightly behind the loss on the swap pair. |

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| PROOF |
| Demonstration of a perfect hedge of the warehousing strategy when par bonds are used as a warehousing instrument.  Variables: *T* ... swap term, *r* ... initial *T*-year swap rate, Δ*r* ... change in the *T*-year swap rate, *N* ... notional value of the swap, *M* ... nominal value of the par bond.  Present value of interest paid in the earlier swap:  Present value of interest received in the later swap (using the par bond’s property according to which the equality between the coupon rate and the discount rate results in the price of the bond equal to its nominal value):  Permanent interest loss on the swap pair (difference of the above formulas):  Purchasing price of the par bond:  Selling price of the par bond:  Number of bonds used for warehousing the swap:  Total capital gain:  It is easy to see that the total capital gain is exactly the same as the permanent loss on the swap pair. |

* the dealer is the recipient of a fixed rate in the earlier swap and is concerned that long-term interest rates may rise during the searching period

Dealer

Money

market

5-year interest

Cash

Interest

5-year bond

3-month LIBOR

Cash

Earlier customer

Bond

market

Bond

lender

5-year bond

the dealer carries out the short sale of a bond whose maturity matches the maturity of the swap (borrowing bonds from the bond lender and selling them on the bond market)

the proceeds from the sale are deposited in the money market and the interest received is used to pay the floating interest in the swap; the dealer is exposed to some basis risk because the deposit rate and the floating swap rate may differ

if the five-year interest rate rises, a capital gain achieved by a lower purchasing price of bonds offsets a permanent income loss on the fixed legs of the matching swaps

**ii) Warehousing with interest rate futures**

* a dealer is the payer of a fixed rate in the earlier swap and is concerned that long-term interest rates may fall during the searching period

Dealer

Money

market

5-year interest

Short position

6-month LIBOR

Long position

Earlier customer

Interest rate

futures

the dealer opens a long position in an interest rate futures contract that profits from falling interest rates; the gain in futures trade offsets a permanent income loss on the fixed legs of the matching swaps

the dealer opens a short position in a short-term interest rate futures contract that makes money on rising futures short-term interest rates; this gain offsets a temporary loss in the floating legs of the swap pair, caused by a higher short-term interest rate paid than received (this loss will be eliminated at the nearest reset date)

an advantage of warehousing with futures contracts is their high liquidity and the small initial payments required by initial margins

a disadvantage of warehousing with futures is considerable basis risk as the maturities of the underlying bonds delivered in futures are usually much longer than maturities of swap contracts

**1.6 Arbitrage strategies using interest rate swaps**

an arbitrage opportunity exists if two interest instruments with similar risky characteristics generate different streams of interest payments

in efficient markets arbitrage opportunities should not exist but in reality price discrepancies occur that can be exploited in arbitrage trades

by exchanging interest payments based on different interest rates, swaps play a crucial role in the integration of financial markets

**i) Liability arbitrage swap**

a swap may reduce the cost of a preferred form of funding if the swap’s user has access to a cheaper but not preferred form of funding

sources of arbitrage opportunities:

* swap rates reflect prevailing market conditions and are not adjusted to variations in the creditworthiness of individual swap counterparties
* subsidized finance
* the different speeds at which individual segments of financial markets respond to the same information
* short-term price anomalies

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| EXAMPLE |
| A corporation can borrow at a fixed interest rate of 8.5 % that is lower relative to prevailing market rates. By putting on a swap in which it receives a market fixed rate of 9.0 %, the corporation can subsidize preferred borrowing in a floating rate.  Corporation  3-year 8.5%  Dealer  (seller)  6-month LIBOR  3-year 9.0%  The corporation created synthetic borrowing at a floating interest rate of Libor – 0.5%. |

**ii) Asset arbitrage swap**

a swap can enhance the return on a preferred form of investment if a swap’s user has access to an asset whose yield is above prevailing market rates

sources of arbitrage opportunities:

* an asset is illiquid or difficult to price, so it has to pay an abnormally higher yield in order to compensate investors for the risks involved (complex securities targeted at narrow groups of investors, illiquid bond issues, etc.)
* the different speeds at which individual segments of financial markets respond to the same information
* short-term price anomalies

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| EXAMPLE |
| An investor holds a bond with a variable coupon of 75 basis points above LIBOR while the floating rate of the same rolled-over period in a swap contract is a flat LIBOR. By putting on a swap, the investor can enhance a fixed income received through the swap. Effectively, he/she creates a synthetic three-year fixed rate asset with an enhanced yield of 75 basis points above the prevailing market conditions of 8%.  Corporation  LIBOR+75bp  Dealer  (seller)  LIBOR  3-year 8.0% |

**1.7 New-issues arbitrage**

**i) Description**

new-issues arbitrage is a strategy that benefits from different credit risk premiums charged on markets with fixed and floating borrowing rates

the mechanism of new-issues arbitrage is similar to the theory of comparative advantage that was proposed by David Ricardo in his explanation of international trade

|  |  |  |
| --- | --- | --- |
|  | Fixed rate | Floating rate |
| Company AA  Company A | 10%  12% | LIBOR + 100bp  LIBOR + 160bp |
| Differential | 200bp | 60bp |
| Arbitrage potential = 200 – 60 = 140bp | | |

terminology:

* the better-rated Company AA can fund itself more cheaply on both markets, so we say that Company AA has an absolute advantage on both markets
* Company AA it can borrow ‘more more cheaply’ at the fixed rate and ‘less more cheaply’ at the floating rate, so we say that Company AA has a comparative advantage at the fixed rate and a comparative disadvantage at the floating rate
* the worse-rated Company A can fund itself more expensively on both markets, so we say that Company A has an absolute disadvantage on both markets
* Company A can borrow ‘less more expensively’ at the floating rate and ‘more more expensively’ at the fixed rate, so we say that Company A has a comparative advantage at the floating rate and a comparative disadvantage at the fixed rate
* an arbitrage potential is the difference between the numerical values of comparative advantages of the two firms; it is the maximum amount of total gain the firms can distribute among themselves
* both firms can benefit from concluding an interest rate coupon swap if their preferred form of borrowing is the interest rate at which they have a comparative disadvantage (Company AA wants to borrow at a floating rate and Company A wants to borrow at a fixed rate)

**ii) Example**

both firms borrow funds on the market in which they have comparative advantage and then swap interest payments in such a way that both save borrowing cost relative to the direct borrowing at the preferred interest rate

one possible arrangement (among an infinite number of others)

Company AA

10.2%

LIBOR

10%

Company A

LIBOR+160bp

* Company AA’s net financial position:

borrowing on the fixed rate market – 10%

paid floating interest in the swap – LIBOR

received fixed interest in the swap + 10.2%

net borrowing cost (10% + LIBOR) – 10.2% = LIBOR – 20bp

net gain LIBOR + 100bp – (LIBOR – 20bp) = 120bp

Company AA succeeded in reducing the preferred floating rate funding by 120 basis points

* Company A’s net financial position:

borrowing on the floating rate market – (LIBOR + 160bp)

paid fixed interest in the swap – 10.2%

received floating interest in the swap + LIBOR

net borrowing cost (LIBOR + 160bp) + 10.2% – LIBOR = 11.8%

net gain 12% – 11.8% = 20bp

Company A succeeded in reducing the preferred fixed rate funding by 20 basis points

in total, both companies saved 120 + 20 = 140bp, which is the size of the available arbitrage potential

Company AA took possession of 86% (= 120/140) and Company A secured 14% (=20/140) of the total arbitrage potential, amounting to 140 basis points

a financially stronger company is likely to acquire a larger share of the arbitrage potential

**iii) Intermediating role of swap dealer**

corporate counterparties are usually reluctant to take on the credit risk of other business firms

a more realistic description of new-issues arbitrage would include a financial institution acting as an intermediary between corporate borrowers with opposite funding needs

the intermediary swap dealer takes part of the total arbitrage potential

example:

Swap dealer

Company AA

10.15%

LIBOR

10%

Company A

LIBOR+160bp

LIBOR

10.25%

distribution of the arbitrage potential:

* Company AA: (LIBOR + 100bp) – (10% + LIBOR – 10.15%) = 115bp
* Company A: 12% – (LIBOR + 160pb + 10.25% – LIBOR) = 15bp
* swap dealer: (10.25% – LIBOR) + (LIBOR – 10.15%) = 10bp

**iv) Credit risk considerations**

evaluation of gains from new-issues arbitrage can be invalidated by a changed credit standing of swap counterparties during the swap

suppose that Company A was downgraded immediately after the swap transaction had been arranged, resulting in the spread widening over the LIBOR from 160bp to 200bp

Company A’s net financial position:

borrowing on the floating rate market – (LIBOR + 200bp)

paid fixed interest in the swap – 10.2%

received floating interest in the swap + LIBOR

net borrowing cost (LIBOR + 200bp) + 10.2% – LIBOR = 12.2%

Company A pays by 20 basis points more in comparison with available direct borrowing at the fixed interest rate of 12%

**1.8 Valuation of coupon swaps**

**i) Formulas**

the valuation of a coupon swap consists of the assessment of the net worth of the two streams of interest payments exchanged through the swap

* value for the swap buyer = PV(floating stream) – PV(fixed stream)
* value for the swap seller = PV(fixed stream) – PV(floating stream)

the valuation of the fixed stream is identical to the valuation of coupon bonds, and the valuation of the floating stream is identical to the valuation of the floating rate notes

valuation formulas can disregard present values of principal amounts that cancel each other out in the net worth of the swap

… principal notional amount of the swap

… fixed interest rate paid in the fixed leg of the swap

… *T*-year yield (yield to maturity of *T*-year coupon bond)

… yield to maturity of *T*-year zero-coupon bond

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| EXAMPLE |
| What is the value of the coupon swap with a notional principal amount of 55 mil USD, a fixed interest rate of 10.63% paid semi-annually, a floating rate also paid semi-annually and 4.5 years remaining to maturity? Assume the current yield for 4.5-maturity is 10.12%. Suppose a flat yield curve.  A flat yield curve implies that the swap rate of 10.12% can be used as a discount rate for calculating the present values of both legs of the swap.  The value of the swap for the swap buyer (the payer of the fixed rate and the receiver of the floating rate):  .  The value of the swap for the swap seller (the payer of the floating rate and the receiver of the fixed rate):  . |

**ii) Bootstrapping of swap rates**

a swap rate is the average of the bid interest rate and the ask interest rate at which swap dealers are prepared to buy (pay the fixed rate) and sell (receive the fixed rate) in coupon swaps

swap rates should be fair, securing zero net present values of newly agreed upon swaps (a positive NPV for one party would mean a negative NPV for the other party)

PV(floating stream) – PV(fixed stream) = 0

swap rates can be viewed as coupons of par bonds

* general pricing formula for a coupon bond whose coupon rate is equal to the swap rate

= PV(coupon payments) + PV(principal amount) = PV(fixed stream of the swap) + PV(principal amount)

* principle of fair valuation of a coupon swap

PV(fixed stream) = PV(floating stream)

= PV(fixed stream) + PV(principal amount) = PV(floating stream) + PV(principal amount)

* property of a floating rate bond

= PV(floating stream) + PV(principal amount) = Principal amount

* the price of a coupon bond with a coupon rate equal to the swap rate is equal to the principal amount of this bond

= Principal amount

the equality between the price of the coupon bond and its principal amount is the key property of the par bond

swap rates that are equal to the yields of par bonds can be used for calculating zero rates (in addition to the traditional bootstrapping of zero rates from a given set of coupon-bearing bonds)

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**2 FORWARD RATE AGREEMENT**

**2.1 Description**

forward rate agreement (FRA) is a financial instrument in which two interest payments must be exchanged at a specific future date, namely

* the FRA rate, based on predetermined interest rate
* the reference rate, based on a future value of an agreed upon market interest rate

FRAs can be seen as one-off interest rate coupon swaps with a delayed start and the exchange of interest taking place at the beginning of the future interest period, called the FRA period

FRA period

FRA is agreed

FRA begins

FRA ends

FRA is settled

interest payments are based on an agreed upon notional principal amount and are executed as a net amount rather a gross amount

an FRA buyer is a party that pays a predetermined FRA rate and receives the reference rate, while an FRA seller is the other party that pays the reference rate and obtains the FRA rate

FRA buyer

FRA rate

FRA seller

(seller)

Reference rate

interest payments are calculated using simple interest in conformity with money market conventions

the net amount is discounted to the beginning of the FRA period when the settlement takes place

* cash flow for the FRA buyer
* cash flow for the FRA seller

*... p*-month FRA rate agreed upon now for the FRA period starting in *t*-months’ time and lasting for *p* months

... *p*-month reference rate which will exist at time *t* when the FRA period starts

... number of days in the FRA period

... notional principal amount of the FRA contract

FRA notation refers to the beginning and end dates of the FRA period

* 6 versus 12 FRA or 6v12 FRA or 6\*12 FRA mean an FRA contract whose FRA period starts in six months’ time and ends in 12 months’ time, so the FRA period lasts for six months
* 0.5v6.5 FRA means an FRA contract whose FRA period starts in two weeks and lasts for six months

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| EXAMPLE |
| On 11 February a company buys from a bank an FRA contract 6v9 with an FRA rate of 12.8% and a notional amount of 1 million EUR. The reference rate is the three-month EURIBOR. Suppose that at the beginning of the FRA period, which is 11 August, the three-month EURIBOR is 11.5 %. Therefore, on 11 August the company pays the bank the amount  As an alternative, let us assume that at the beginning of the FRA period the three-month EURIBOR is 13.5 %. So, on 11 August the company receives from the bank the amount |

**2.2 Applications of FRA**

**i) Speculative trades**

an FRA can be used to speculate whether the reference rate will be higher or lower than the FRA rate at the beginning of the FRA period

* when money market interest rates are expected to rise, speculators want to buy FRA contracts with the aim to benefit from the higher received reference rate than the paid FRA rate
* when money market interest rates are expected to fall, speculators want to sell FRA contracts with the aim to benefit from the higher received FRA rate than the paid reference rate

speculative gain can be achieved without upfront investment because the FRA’s principal amount is notional

**ii) Hedging trades**

* locking in short-term borrowing interest rate

Borrower

Money

market

LIBOR

FRA dealer

LIBOR + spread

FRA rate

|  |
| --- |
| EXAMPLE |
| A company rolls over a three-month bank loan in which the interest rate is re-fixed at the prevailing three-month LIBOR plus a given margin. The next rollover date is in two months’ time. The company wants to be hedged against an interest rate increase, so it buys a 2v5 FRA.  The net interest rate position at the beginning of the FRA contract:  company pays a current three-month LIBOR + margin to lending bank  company pays an agreed upon FRA rate to the FRA counterparty  company receives a current three-month LIBOR from the FRA counterparty  The net borrowing cost is the FRA rate + margin, which is immune to interest rate changes. |

* locking in short-term deposit interest rate

Lender

Money market

LIBOR

FRA dealer

LIBOR + spread

FRA rate

|  |
| --- |
| EXAMPLE |
| In two weeks’ time, a company expects to make a six-month deposit yielding a six-month LIBOR plus the spread. It is concerned that the interest rate may fall in the meantime. To be hedged against the interest rate fall, the company sells a 0.5v6.5 FRA.  The net interest rate position at the beginning of the FRA contract:  company receives a current six-month LIBOR plus the spread from the deposit institution  company receives an agreed upon FRA rate from the FRA counterparty  company pays a six-month LIBOR to the FRA counterparty  The net return is the FRA rate + spread, which is immune to interest rate changes. |

**2.3 FRA strip**

**i) Description**

FRA strip is a synthetic FRA contract created from a series of consecutive FRAs

* FRA contracts 3v5, 5v8 and 8v9 can make up a synthetic 3v9 FRA strip
* three-month loan (viewed as 0v3 FRA) in combination with 3v5, 5v8 and 8v9 FRAs can make up a synthetic nine-month loan (viewed as synthetic 0v9 FRA)

**ii) Rules of construction**

the construction of an FRA strip is illustrated by the sequence of 3v5, 5v8 and 8v9 FRAs, which make up an FRA strip 3v9

* the amounts of notional principals of the constituting FRAs have to take into account the FRA rates of the preceding FRA contracts in accordance with the following formulas

the amounts of the notional principal are known in advance because the FRA rates are also known at the time when the strip is constructed

* FRAs settlements must be combined with borrowing and lending on the money market in such a way that the resulting cash flow replicates the cash flow of the constructed FRA strip

|  |  |  |
| --- | --- | --- |
| Time | Actions | Net cash flow |
| 0 | Buy 3v5, 5v8 and 8v9 FRA contracts | 0 |
| 3 | Settle the 3v5 FRA  Lend *M* for the 3v9 FRA period  Borrow the balance for the 3v5 FRA period | 0 |
| 5 | Settle the 5v8 FRA  Repay the 3v5 loan  Borrow the balance for the 5v8 FRA period | 0 |
| 8 | Settle the 8v9 FRA  Repay the 5v8 loan  Borrow the balance for the 8v9 FRA period | 0 |
| 9 | Repay the 8v9 loan  Terminate the 3v9 investment | Accumulated value of the settlement amount of the 3v9 FRA strip |

* no arbitrage condition between the synthetic 3v9 FRA strip and the constituting FRAs implies interest rate parity conditions

|  |
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| PROOF |
| Verify the rules for the construction of a 3v9 FRA strip using the FRA contracts 3v6 and 6v9. The FRA period of both constituting FRAs is 91 days. denotes the required notional amount of the FRA strip.  Actions taken at time   1. Buy the 3v6 FRA in a notional amount of 2. Buy the 6v9 FRA in a nominal amount of   There is no initial cash flow induced by entering constituting FRAs.  Actions taken at time   1. Settle the 3v6 FRA, which results in the cash flow 2. Lend for six months (this period is equal to the term of the 3v9 FRA strip) at the prevailing six-month market rate . 3. Refinance the balance with a three-month loan at the prevailing three-month interest rate . The size of the loan is   Actions taken at time   1. Settle the 6v9 FRA, which results in the cash flow 2. Repay the previous three-month loan, which results in the cash outflow in the amount of 3. Refinance the balance with a new three-month loan at the prevailing three-month interest rate . The size of the loan is   Actions taken at time   1. Repay the previous three-month loan, which results in the cash outflow in the amount of 2. Terminate the six-month deposit, which results in the cash inflow in the amount of   The net amount of all cash flows is  Let’s apply the interest rate parity conditions between the FRA rate of the FRA strip 3v9 and the FRA rates of constituting FRAs 3v6 and 6v9  We get  This is the settlement value of the FRA strip 3v9, accumulated over its six-month FRA period at the six-month market interest rate which prevailed at the time of the strip’s settlement. |

|  |
| --- |
| EXAMPLE |
| Construct a synthetic nine-month borrowing rate from   1. the current three-month LIBOR of 8.2% and a 92-day lending period, 2. 3v6 FRA with an FRA rate of 8.7% and a 91-day FRA period, 3. 6v9 FRA with an FRA rate of 9.5% and 91-day FRA period.   FRA contracts are settled against a market three-month LIBOR.  Actions taken at time   1. Borrow cash for three months at an amount of one monetary unit. 2. Buy the 3v6 FRA with a notional amount 3. Buy the 6v9 FRA with a notional amount   Actions taken at time   1. Repay the three-month loan 2. Settle the 3v6 FRA 3. Refinance the balance by taking a new three-month loan   Actions taken at time   1. Repay the three-month loan 2. Settle the 6v9 FRA 3. Refinance the balance by taking a new three-month loan   Actions taken at time   1. Repay the three-month loan   The effective nine-month borrowing rate can be determined from the equation  The same effective rate can be determined from the interest rate parity between the FRA strip and the constituting FRA contracts.  .  We get |

**2.4 Price links with coupon swaps**

both FRA strips and coupon swaps can be used for simulating money market borrowing of a given maturity

* an FRA strip composed of a three-month loan, 3v6 FRA, 6v9 FRA and 9v12 FRA creates a synthetic 12-month loan
* a 12-month loan can be also achieved by a combination of a one-year coupon swap with a three-month floating leg

the net cash flow consists of the initial cash inflow and the terminal cash outflow that is tantamount to

* a 12-month loan with a principal amount of and a 12-month interest rate
* an FRA strip 0v12

remarks

* the solid arrows represent the cash flow of the interest rate coupon swap
* the dashed arrows represent investments in the money market
* for the sake of simplicity, the interest rates are not adjusted for the number of days in the interest period

interest rate parity condition

imperfections may arise due to the different conventions used in money markets (applied to FRA contracts) and capital markets (applied to swaps)

|  |
| --- |
| EXAMPLE |
| Determine the fixed rate of a one-year interest rate coupon swap, given the following money market and FRA rates:  three-month LIBOR: 14.1%, 91 days,  3v6 FRA: 12.4%, 91 days,  6v9 FRA: 11.6%, 91 days,  9v12 FRA: 11.2%, 92 days.  On efficient markets, the fixed rate of the one-year coupon swap satisfies the interest rate parity condition  The one-year swap rate should be quoted at a price of 12.9 %. |

**2.5 Price links with interest rate futures contracts**

**i) Similarities and differences**

an FRA contract is a negotiable equivalent to a highly standardized short-term interest rate futures contract

a financial settlement of both an FRA and an interest rate futures is based on the difference between a predetermined (known-in-advance) and a future (unknown-in-advance) short-term interest rate

both FRA and futures contracts are settled at the beginning of a given future interest rate period (FRA period versus delivery date)

* pay-off for the buyer (long) of a three-month interest rate futures with the delivery at time *T*

opening futures price

closing futures price

implied opening three-month rate (known-in-advance)

implied closing three-month rate (unknown-in-advance)

size of futures contract

* pay-off for the buyer of a three-month FRA with FRA period starting at time *T*

*... FRA* rate (known-in-advance)

... 3-month interest rate (unknown-in-advance)

approximate equality between the FRA rate and the implied futures interest rate

* an FRA period and the period of the futures’ underlying deposit start at the same time and have the same length ()
* the notional amount of the FRA is equal to the size of the futures contract
* both contracts use the same reference rate (3-month LIBOR prevailing at time )
* the quantitative effect of discounting must be ignored (the settlement amount of the FRA is based on discounted values while the settlement of a futures contract is based on undiscounted values)

opposite meaning of long and short positions

* a long position in interest rate futures means paying the unknown-in-advance rate and receiving the known-in-advance rate; the meaning of the long or the buyer is opposite in FRA contracts
* a short position in interest rate futures means receiving the unknown-in-advance rate and paying the known-in-advance rate; the meaning of the short or the seller is opposite in FRA contracts

**ii) Futures-linked FRA contracts**

futures-linked FRA is an FRA contract that derives its price (FRA rate) from the quote of a financially equivalent interest rate futures contract

arbitrage opportunities would exist if the two linked contracts are substantially different

|  |
| --- |
| EXAMPLE |
| Today is 18 March. The following prices are quoted for the ST3 futures contract whose underlying asset is a three-month LIBOR sterling deposit:  June ST3: 91.75 (implied rate 8.25%), delivery day 17 June  September ST3: 91.50 (implied rate 8.50%), delivery day 17 September  December ST3: 91.25 (implied rate 8.75%), delivery day 17 December  Which FRA contracts settled at the three-month LIBOR are likely to be priced close to the above implied interest rates?  The June ST3 covers the three-month period starting on 18 June and ending on 18 September. So, on 18 March the 3v6 FRA, which also starts on 18 June and ends on 18 September should have an FRA rate close to 8.25%.  For the same reason, on 18 March the 6v9 FRA, which starts on 18 September and ends on 18 December, should have an FRA rate close to 8.50%.  For the same reason, on 18 March the 9v12 FRA, which starts on 18 December and ends on 18 March should have an FRA rate close to 8.75%. |

practical hitches that complicate the pricing of futures-linked FRA contracts

* different conventions in futures markets (ACT/360) and in FRA markets (ACT/365)
* notional amounts of futures-linked FRA contracts can only be multiples of a rounded number of futures contracts
* slight differences in dates for fixing the settlement rates in futures and FRA contracts

table of all futures-linked FRA contracts (derived from March, June and December three-month interest rate futures); column numbers denote the beginning of the FRA period, and the row numbers represent the end of the FRA period

🟊 futures-linked FRA contracts

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
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**iii) Futures-linked FRA strips**

FRA strip is a FRA contract created synthetically from a series of consecutive FRAs

futures-linked FRA strip is a strip created synthetically from futures-linked FRA contracts; it is priced in accordance with an interest rate parity condition

|  |
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| EXAMPLE |
| Today is 18 March. What futures-linked FRA strips can be created and what should their FRA rates be, given the futures-linked FRAs from the previous example?  Recapitulation of available futures-linked FRA contracts:  3v6 FRA: FRA rate of 8.25% and FRA period of 92 days  6v9 FRA: FRA rate of 8.50% and FRA period of 91 days  9v12 FRA: FRA rate of 8.75% and FRA period of 90 days.  With the 3v6, 6v9 and 9v12 the following strips can be crated:  3v6 and 6v9 FRAs form a 3v9 FRA strip (number of days: 92 + 91 = 183)  6v9 and 9v12 FRAs form a 6v12 FRA strip (number of days: 91 + 90 = 181)  3v6, 6v9 and 9v12 FRAs form a 3v12 FRA strip (number of days: 92 + 91 + 90 = 273)  The FRA rate for the 3v9 FRA strip:  The FRA rate for the 6v12 FRA strip:  The FRA rate for the 3v12 FRA strip: |

table of all futures-linked FRA contracts (derived from March, June and December three-month interest rate futures); column numbers denote the beginning of the FRA period, and the row numbers represent the end of the FRA period

🟊 futures-linked FRA contracts

◾ futures-linked FRA strips

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
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**iv) Linear interpolation ‘same start, different length’**

the family of futures-linked FRAs can be expanded if we can reasonably estimate prices of other FRAs using an appropriate method of prediction

linear interpolation is a possible way to solve this approximation problem

the method ‘same start, different length’ can be applied when an unknown FRA rate is flanked by two known futures-linked FRA rates that start at the same future time as the unknown FRA rate but vary in the length of their FRA periods

an unknown FRA rate 3v8 is flanked by two known futures-linked FRA rates 3v6 and 3v9

we ask the question: what will the five-month FRA rate starting in three months’ time likely to be, given that the three-month and six-month FRA rates both start in three months’ time

number of days in the FRA period

property of similar triangles

|  |
| --- |
| EXAMPLE |
| Determine FRA rates of the 3v8 and 6v11 FRA contracts, given the FRA rates of futures-linked FRA contracts 3v6, 3v9, 6v9 and 6v12 from the previous example. Assume that the FRA periods of both 3v8 and 6v11 FRA contracts have 153 days.  The FRA contract 3v8 is flanked by the following futures-linked FRA contracts, satisfying the property ‘same start, different length’:  3v6 FRA: FRA rate of 8.25% and FRA period of 92 days,  3v9 FRA: FRA rate of 8.46% and FRA period of 183 days.  Linear interpolation results in  The FRA contract 6v11 is flanked by the following futures-linked FRA contracts, satisfying the property ‘same start, different length’:  6v9 FRA: FRA rate of 8.50% and FRA period of 91 days,  6v12 FRA: FRA rate of 8.72% and FRA period of 181 days.  Linear interpolation results in |

table of all futures-linked FRA contracts (derived from March, June and December three-month interest rate futures); column numbers denote the beginning of the FRA period, and the row numbers represent the end of the FRA period

🟊 futures-linked FRA contracts

◾ futures-linked FRA strips

**→** interpolations ‘same start, different length’

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **3** | 🟊 | **→** | **→** | ◾ | **→** | **→** | ◾ |
| **4** |  |  |  |  |  |  |  |
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| **6** |  |  |  | 🟊 | **→** | **→** | ◾ |
| **7** |  |  |  |  |  |  |  |
| **8** |  |  |  |  |  |  |  |
| **9** |  |  |  |  |  |  | 🟊 |

**v) Linear interpolation ‘same length, different start’**

the method ‘same length, different start’ can be used if an unknown FRA rate is flanked by two known futures-linked FRA rates that have the same length of FRA period as the unknown FRA rate but vary in the future time of their start

example: an unknown FRA rate 5v10 is flanked by two known futures-linked FRA rates 3v8 and 6v11

we ask the question: what will the five-month FRA rate starting in five months’ time likely be, given that the five-month FRA rates start in three months’ time and six months’ time

number of days to the start of FRA period

property of similar triangles

|  |
| --- |
| EXAMPLE |
| Determine the FRA rate of the 5v10 FRA contract, given the FRA rates of futures-linked FRA contracts 3v8 and 6v11 from the previous example. Assume that the number of days to the start of the FRAs 3v8, 5v10 and 6v11are 92, 153 and 184 days respectively.  The FRA contract 5v10 is flanked by the following futures-linked FRA contracts, satisfying the property ‘same length, different start’:  3v8 FRA: FRA rate of 8.39% and 92 days to its start,  6v11 FRA: FRA rate of 8.65% and 184 days to its start.  Linear interpolation results in |

table of all futures-linked FRA contracts (derived from March, June and December three-month interest rate futures)

🟊 futures-linked FRA contract

◾ futures-linked FRA strip

**→** same start, different length

**↘** same length, different start

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **3** | 🟊 | **→** | **→** | ◾ | **→** | **→** | ◾ |
| **4** |  | **↘** | **↘** | **↘** | **↘** |  |  |
| **5** |  |  | **↘** | **↘** | **↘** | **↘** |  |
| **6** |  |  |  | 🟊 | **→** | **→** | ◾ |
| **7** |  |  |  |  | **↘** |  |  |
| **8** |  |  |  |  |  | **↘** |  |
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**3 CURRENCY SWAP**

**3.1 Exchange rate conventions**

exchange rate is the price of one currency in terms of other currency (1 EUR = 1.25 USD, 1 USD = 0.8 EUR, 1 GBP = 1.32 EUR)

different roles of currencies in a currency pair

* base currency (BAC) is the measured currency; BAC is represented by one unit in the exchange rate quotation
* variable currency (VAC) is the measuring currency; VAC is represented by the number of units in the exchange rate quotation

quotation conventions – illustrated by the exchange rate 1 EUR = 1.25 USD (1.25 dollars per one euro)

* fraction convention (preferred notation in this course): 1.25 USD/EUR (x VAC/BAC); separating slash is seen as a fraction denoting the number of units of the numerator (dollars) per one unit of the denominator (euro)
* practitioners’ convention: 1.25 EUR/USD, 1.25 EURUSD ( x BAC/VAC, x BACVAC)

national practices

* indirect quotation uses the foreign currency as the base currency and the domestic currency as the variable currency (the number of units of domestic currency per one unit of foreign currency)
* direct quotation uses the domestic currency as the base currency and the foreign currency as the variable currency (the number of units of foreign currency per one unit of domestic currency)
* each of these two quotations is the reciprocal value of the other

1.25 USD/EUR is equivalent to 1/1.25 EUR/USD = 0.8 EUR/USD

ambiguous measurement of larger percentage exchange rate changes

suppose the euro strengthened from the value of 1.25 dollars to the value of 1.5 dollars per one euro

* the euro is the base currency and the dollar is the variable currency ⇒ the exchange rate 1.25 USD/EUR changes to a new value of 1.50 USD/EUR
* the euro is the variable currency and the dollar is the base currency ⇒ the exchange rate 1/1.25 = 0.8 EUR/USD changes to a new value of 1/1.5 = 0.67 EUR/USD
* the smaller the exchange rate change, the smaller is the difference between the results of the two measures

|  |
| --- |
| PROOF |
| For small exchange rate changes, the choice of the base and the variable currency in the currency pair has little impact on the measurement of percentage change of an exchange rate.  Let H and F denote the currency codes for the home and foreign currencies respectively. Let and denote percentage changes of the exchange rate between the home and foreign currencies, based on the indirect quotation and the direct quotation respectively.  We have  The relationship between the two percentage small changes is as follows:  The last adjustment uses the approximation formula which holds for the values of *x* close to zero. |

**3.2 Covered interest rate parity**

covered interest rate parity is an equation that is built on the assumption of the unfeasibility of the trade strategy, called cash-and-carry transaction

**i) Cash-and-carry transaction**

variables

* *S* – spot exchange rate; this is a rate agreed upon now for outright purchase or sale of one currency for another currency (spot transactions can be settled later but usually not beyond two days)
* *F* – forward exchange rate; this is a rate agreed upon today for trades that will take place at a certain future point in time
* – money market interest rate prevailing in the base currency’s area
* – money market interest rate prevailing in the variable currency’s area
* *T* – duration of the cash-and-carry transaction

description of cash-and carry-transaction

* in a currency pair, the euro is the base currency and the dollar is the variable currency
* horizontal arrows represent the borrowing and lending of the participating currencies
* vertical arrows represent conversions between the participating currencies.

USD

EUR

**①**

**②**

**③**

**⑤**

**④**

* 1. borrow one unit of euro
  2. convert one euro to 𝑆 units of dollar at a spot exchange rate 𝑆 and arrange the forward conversion of dollars back to euros at a forward exchange rate 𝐹
  3. deposit *S* units of dollars at a dollar interest rate to get dollars when the dollar deposit is terminated
  4. convert the dollar deposit balance back to euros at an agreed upon forward exchange rate 𝐹 to get euros
  5. repay the euro loan plus euro interest

cash-and-carry transaction should generate neither a profit nor a loss

covered interest rate parity

**ii) Forward discount and premium**

alternative representation of covered interest rate parity

the last adjustment uses the approximation formulas for the values close to zero:

and

implications

* a higher interest-yielding currency must have a weaker forward rate against the spot rate

⇒ a currency with a weaker forward rate against the spot rate is sold or bought at a forward discount

* a lower interest-yielding currency must have a stronger forward rate against the spot rate

⇒ a currency with a stronger forward rate against the spot rate is sold or bought at a forward premium

economic rationale

a profit from a higher interest rate must be offset by an exchange rate loss during the reverse currency conversion, otherwise arbitrage profit could be earned

**iii) Uncovered interest rate parity**

an expectation hypothesis asserts that the current forward exchange rate is the best estimate of the future spot exchange rate

... current forward rate (agreed upon for time *T*)

... today’s expectation about the spot exchange rate prevailing at time *T*

uncovered interest rate parity is a version of covered interest rate parity in which the forward rate is replaced by the expected spot rate

interpretation

* the anticipated exchange rate appreciation or depreciation is equal to the current differential between interest rates prevailing in participating currency areas
* markets expect an appreciation for a lower yielding currency and deprecation for a higher yielding currency

**iv) Non-deliverable forward**

non-deliverable forward is a trading strategy that profits from the inconsistency between covered and uncovered interest rate parities that may happen in fixed exchange rate regimes

economic rationale

* fixed exchange rate between the Czech koruna (CZK) and the German mark (DEM)
* higher interest rates in the Czech Republic than in Germany
* current CZK/DEM forward rate is weaker than the current and future CZK/DEM spot rates according to the covered interest rate parity and the fixed exchange rate regime (CZK is weaker on the forward market than on the spot market)

a risk-free profit can be achieved using the following strategy

sell one mark forward for korunas (no initial investment)

i) borrow 1 DEM

ii) immediately deliver 1 DEM to the forward contract in exchange for receiving korunas

iii) immediately sell korunas on the spot market in exchange for receiving DEM

iv) immediately repay the 1 DEM loan (interest can be neglected)

size of the arbitrage profit = DEM (it is a positive number)

a bank may arrange all future transactions simultaneously and share the arbitrage profit with the speculator

practical complications that make the future spot exchange rate less predictable: fluctuation bands in the fixed exchange regime, peg to a basket of currencies

**3.3 Related currency instruments**

**i) Outright forward**

outright forward is an agreement concluded today about the exchange of principal amounts denominated in two different currencies taking place at some future date

the future exchange will take place at the forward exchange rate (usually different from the spot rate)

there is no exchange of interest amounts but the differential between the spot and forward rates reflects differences in interest rates (in line with covered interest rate parity)

EA Bank

US Bank

USD

EUR

**ii) Foreign exchange (forex) swap**

foreign exchange swap is an agreement about the temporary exchange of principal amounts denominated in two different currencies taking place both at the start and maturity of the swap deal

the initial exchange is made at the prevailing spot exchange rate and the re-exchange is made at the forward exchange rate prevailing at the start of the swap; because spot and forward exchange rates are usually different, the amounts of principal exchanged at maturity differ from those exchanged at the start

there is no exchange of interest amounts but the differential between the spot and forward rates reflects differences in interest rates (in line with covered interest rate parity)

EA Bank

US Bank

USD

EUR

USD

EUR

a forward swap can be created synthetically as a combination of a spot deal and an outright forward

EA Bank

US Bank

USD

EUR

EA Bank

US Bank

USD

EUR

**+**

**iii) Currency swap**

currency swap commits two counterparties to exchange

* two streams of interest payments in different currencies over an agreed upon period
* at the end of the contract corresponding principal amounts at an exchange rate agreed upon at the start of the contract

USD interest

EA Bank

US Bank

USD

EUR

EUR interest

principal amounts can be exchanged at the start of the swap as well; in that case the re-exchange of principal amounts will be at the original exchange rate (principal amounts exchanged at maturity will be the same as those exchanged at the start of the swap)

an agreed upon exchange rate is usually the spot exchange rate prevailing at the start of the swap but not necessarily; it may be subject to negotiation between the swap counterparties

a currency swap is not a derivative instrument in the strict sense because there is an eventual movement of principal amounts (a derivative is an instrument whose performance is entirely derived from the performance of an underlying asset without the need to buy or sell this asset)

a currency swap can be constructed with no exchange of principal even at maturity; in that case the settlement includes compensation for the payer of the currency that appreciates over the life of the swap

types of currency swaps

* currency coupon swap consists of an exchange of interest streams both of which are derived from a fixed rate of interest
* cross-currency swap consists of an exchange of interest streams of which at least one is derived from a floating rate of interest
* a cross-currency coupon swap is a fixed-against-floating swap
* a cross-currency basis swap is a floating-against-floating swap
* cocktail swap denotes a combination of currency swaps denominated in different currencies, which usually form a hedged structure

Customer

Swap dealer

Customer

Customer

JPY interest

JPY interest

EUR interest

EUR interest

USD LIBOR

USD LIBOR

**iv) Precursors of currency swaps**

parallel loans consist of the simultaneous provision of two loans denominated in two different currencies

both loans are treated as two separate transactions; no one party has the right to get rid of its obligations if the other party defaults

the transactions appear on the balance sheet like conventional loans

back-to-back loans consist of the simultaneous provision of two loans denominated in two different currencies with the right to be compensated for the default but still with two separate sets of rights and obligations

**3.4 Risk management with currency swaps**

currency swaps create an exposure to changes in exchange rates and interest rates so they can be used for hedging and assuming exchange rate risk, interest rate risk or both

currency swaps are used predominantly to hedge currency risk; they are not convenient tool for taking currency because low liquidity makes it difficult to open and close speculative positions

currency swaps can be used for hedging and taking exchange rate risk

**i) Borrowing in foreign currency**

* currency coupon swap combined with fixed interest foreign liability

a currency coupon swap hedges the fixed interest foreign borrowing against exchange rate appreciation and simultaneously protects against rising domestic interest rates

in the swap the borrower pays the fixed interest in the domestic currency from earnings generated by its domestic business and receives the fixed interest in the foreign currency, which is used to service the fixed interest foreign debt

Borrower

Swap dealer

domestic currency

fixed interest

foreign currency

fixed interest

domestic currency

revenue

foreign currency

fixed interest

domestic currency principal

foreign currency principal

foreign currency

debt repayment

* cross-currency coupon swap combined with fixed interest foreign liability

a cross-currency coupon swap hedges the fixed interest foreign borrowing against exchange rate appreciation and simultaneously allows to benefit from falling domestic interest rates

in the swap the borrower pays the floating interest in the domestic currency from earnings generated by its domestic business, and receives the fixed interest in the foreign currency, which is used to service the fixed interest foreign debt

Borrower

Swap dealer

domestic currency

floating interest

foreign currency

fixed interest

domestic currency

revenue

foreign currency

fixed interest

domestic currency principal

foreign currency principal

foreign currency

debt repayment

* cross-currency coupon swap combined with floating interest foreign liability

a cross-currency coupon swap hedges floating interest foreign borrowing against exchange rate appreciation and simultaneously protects against rising domestic interest rates

in the swap the borrower pays the fixed interest in the domestic currency from earnings generated by its domestic business and receives the floating interest in the foreign currency, which is used to service the floating interest foreign debt

Borrower

Swap dealer

domestic currency

fixed interest

foreign currency

floating interest

domestic currency

revenue

foreign currency

floating interest

domestic currency principal

foreign currency principal

foreign currency

debt repayment

* cross-currency basis swap combined with floating interest foreign liability

a cross-currency basis swap hedges floating interest foreign borrowing against exchange rate appreciation and simultaneously allows to benefit from falling domestic currency interest rates

in the swap the borrower pays the fixed interest in the domestic currency from earnings generated by its domestic business, and receives the floating interest in the foreign currency, which is used to service the floating interest foreign debt

Borrower

Swap dealer

domestic currency

floating interest

foreign currency

floating interest

domestic currency

revenue

foreign currency

floating interest

domestic currency principal

foreign currency principal

foreign currency

debt repayment

**ii) Investing in foreign currency**

a currency coupon swap hedges fixed interest foreign investment against exchange rate depreciation and simultaneously protects against falling domestic currency interest rates

the investor receives the fixed interest in domestic currency through the swap and in exchange he pays the fixed interest in foreign currency, which is funded by the interest received on its foreign borrowing

Borrower

Swap dealer

domestic currency

fixed interest

foreign currency

fixed interest

domestic currency

expenditures

foreign currency

fixed interest

domestic currency principal

foreign currency principal

foreign currency

investment repatriation

analogous diagrams can be made for the following trading strategies

* hedging fixed interest foreign investment and benefiting from rising domestic currency interest rates
* hedging floating interest foreign investment and protecting against falling domestic currency interest rates
* hedging floating interest foreign investment and benefiting from rising domestic currency interest rates

**3.5 New-issues arbitrage**

new-issues arbitrage using a currency swap involves cash flows in two different currencies that must be put on a comparable basis

the comparison is achieved by using the equation of uncovered interest rate parity

basis conversion factor (BCF) is the number of basis points in the variable currency, equivalent to one basis point in the base currency (i.e. one euro basis point is equivalent to 0.9 dollar basis points)

**i) Example**

|  |  |  |
| --- | --- | --- |
|  | EUR rate | USD rate |
| Company A  Company B | 6.5%  7.0% | 4.0%  5.2% |
| Differential | EUR 50bp | USD 120bp |
| Arbitrage potential = USD 120bp – EUR 50bp | | |

alternative evaluations of the arbitrage potential, assuming EUR 1bp = USD 0.9bp

* USD 120bp – EUR 50bp = USD 120bp – USD (50 × 0.9)bp = USD 75bp
* USD 120bp – EUR 50bp = EUR (120/0.9)bp – EUR 50bp = EUR 83.33bp

terminology:

* Company A has higher borrowing costs and Company B has lower borrowing costs on both markets, so Company A has an absolute advantage and Company B has an absolute disadvantage on both euro and dollar markets
* Company A’s interest rate differential is greater on the dollar market (it can borrow there ‘more less cheaply’), so Company A has a comparative advantage on the dollar market and a comparative disadvantage on the euro market
* Company B’s interest rate differential is smaller on the euro market (it can borrow there ‘less more expensively’), so Company B has a comparative advantage on the euro market and a comparative disadvantage on the dollar market
* both firms can benefit from putting on a currency swap if they want to obtain financing on the market on which they have a comparative disadvantage (Company A needs to borrow euros while Company B needs to borrow dollars)

possible arrangement of new-issues arbitrage

* both firms borrow funds on the markets where they have a comparative advantage and then swap interest payments in such a way that both of them save borrowing costs relative to direct borrowing without the swap
* assuming the current exchange rate USD/EUR = 1.3, Company A issues dollar denominated bonds in an amount of 130 million USD and Company B issues euro denominated bonds in an amount of 100 million EUR

Euro

market

100M EUR

Company B

Company A

Dollar

market

100M EUR

130M USD

4% USD

4% USD

6% EUR

130M USD

7% EUR

100M EUR

130M USD

100M EUR

130M USD

* Company A’s net financial position:

interest paid on dollar market – 4% USD

interest received in the swap + 4% USD

interest paid in the swap – 6 % EUR

net borrowing cost – 4% USD + 4% USD – 6% EUR = – 6% EUR

net gain 6.5% EUR – 6.0% EUR = EUR 50bp

Company A reduced its cost of euro funding by 50 euro basis points

* Company B’s net financial position:

interest paid on euro market – 7% EUR

interest received in the swap + 6% EUR

interest paid in the swap – 4% USD

net borrowing cost – 4% USD – 1% EUR = – 4% USD – 0.9% USD

= – 4.9% USD

net borrowing cost 5.2% USD – 4.9% USD = USD 30bp

Company B reduced its cost of dollar funding by 30 dollar basis points

combined cost reduction:

euro basis points = 50 (A) + 30/0.9 (B) = 83.3 (A+B) = arbitrage potential expressed in euros

dollar basis points = 50×0.9 (A) + 30 (B) = 75 (A+B) = arbitrage potential expressed in dollars

**3.6 Warehousing of currency swaps**

warehousing is a hedging technique that aims to eliminate or mitigate risk exposures stemming from concluding matching currency swaps in different moments of time

warehousing of a currency swap deals with hedging both interest rate and exchange rate risks

warehousing scheme for a cross-currency coupon swap with initial exchange of principal amounts

Earlier

customer

Swap dealer

Money

market

Euro

market

5-year

EUR interest

USD loan

USD overnight rate

5-year bond

6-month

USD LIBOR

EUR cash

EUR principal

USD principal

USD principal

EUR principal

risk exposures faced by the swap dealer during the searching time for the matching swap

* a fall in long-term euro interest rates ⇒ for a period of five years a five-year euro interest rate received from a later currency swap would be lower than the five-year euro interest rate paid in the earlier currency swap
* a rise in short-term dollar interest rates ⇒ until the next USD LIBOR refixing date a higher dollar interest would be paid in the floating leg of the later swap compared with the dollar interest received in the floating leg of the earlier swap
* a depreciation of the dollar against the euro ⇒ at maturity of the swap pair a higher dollar principal amount would be paid in the later swap compared with the dollar principal amount received in the earlier swap, while the euro principal amounts would be the same in both swaps

dollar weakened from 1 EUR = 1.1 USD to 1 EUR = 1.2 USD

Earlier

customer

Swap dealer

Later

customer

110 USD principal

100 EUR principal

120 USD principal

100 EUR principal

warehousing strategy

* buying five-year euro bonds for euro funds received in the initial exchange of principals in the earlier currency swap
* a fall in euro interest rates will increase the price of the euro denominated bonds and the resulting capital gain, realized when the bonds are sold, will compensate the dealer for the permanent loss in euro interest received and paid in the swap pair
* a depreciation of the dollar will increase the dollar value of the euro denominated bonds that will offset the mismatch between dollar amounts received and paid at maturity of the swap pair
* borrowing dollars to fund the purchase of euros in the initial exchange of principal amounts in the earlier currency swap (using overnight money because of the unknown length of the warehousing period)
* the funding operation entails the basis risk caused by non-parallel movements of the overnight rate paid and the six-month LIBOR received in the swap

**3.7 Valuation of currency swaps**

the value of a currency swap is equal to the difference between net present values of the future cash flows to be paid and received in the swap

different currency denominations of constituting cash flows are converted to a common basis by using the market exchange rate

* swap value in units of currency A acting as the base currency
* swap value in units of currency B acting as the variable currency

the swap value based on current market interest and exchange rates should be zero when the swap is originated

the swap may acquire non-zero value as a result of market movements in the underlying interest and exchange rate movements

|  |
| --- |
| EXAMPLE |
| A currency coupon swap has three years remaining to maturity. It was designed as a swap with the final exchange of principal amounts of 150 mil USD and 100 mil EUR. The dollar interest stream pays a fixed rate of 4 % and the euro interest stream pays a fixed rate of 10 %. The current three-year dollar swap rate is 5 % and the current three-year euro swap rate is 9 %. What is the market dollar value of the swap, assuming the current dollar-euro exchange rate 1.22 USD/EUR?  Current value of the dollar leg of the swap:  Current value of the euro leg of the swap:  Current dollar value of the swap: |

**3.8 Two case studies in using currency swaps**

**i) Issuance of eurobonds in a high-yield currency**

economic context: In the first half of the 1990s, the Czech monetary authority was surprised by exchange rate appreciation caused by a surge of activity of foreign entities (banks, municipalities) in issuing bonds denominated in Czech koruna. The main reason for this unexpected development was the reduction of the borrowing cost in domestic currencies (German marks, Belgian francs) by exploiting large interest rate differentials vis-à-vis the transition Czech economy.

1. German investors who are interested in high-yield Czech assets buy CZK for DEM, putting pressure on the CZK/DEM exchange rate, so the Czech koruna appreciates.
2. German investors use CZK for buying high-yield bonds denominated in CZK from a German issuing entity.
3. German issuer arranges a cross-currency coupon swap with a swap house with the aim of having a net liability in DEM at a lower cost in comparison to direct borrowing on the German market.
4. The swap house borrows DEM on the foreign exchange market that are sold to the German issuing entity in exchange for receiving CZK.
5. The swap house buys high-yield Czech assets (buys Treasury bills, extends credits, opens bank deposits).

12 % CZK

8 % CZK

(LIBOR–1%) DEM

Swap

house

CZK

DEM

Czech

assets

**(C)**

CZK

**(E)**

8 % CZK

German

investor

Eurokoruna

issuer

CZK

DEM

**(A)**

CZK

**(B)**

Czech bank

LIBOR DEM

DEM

**(D)**

Money

market

Summary

* foreign investors access a high yield with limited exchange rate risk by receiving 8% in CZK
* German issuer borrows more cheaply by paying 1% below German mark LIBOR
* swap house receives 4% CZK – 1% DEM and contains exchange rate risk by employing additional hedging operations

**ii) External borrowing avoiding appreciation pressures**

economic context: The Czech Ministry of Finance decided to borrow more cheaply in foreign markets by issuing euro denominated bonds. Conversions of euros in Czech korunas would cause an undesirable appreciation of the CZK/EUR exchange rate that was to be avoided.

1. The Czech government borrows EUR by issuing foreign bonds denominated in EUR. It will have to pay EUR interest from this borrowing.
2. The swap house arranges a cross-currency swap with the Czech government, which transforms the government’s net liability into domestic currency that can be repaid by revenues collected in the domestic currency.
3. The swap house invests EUR received in the swap back into the foreign exchange market. There are no conversions of EUR into CZK on the foreign exchange market, so there is no appreciation impact on the CZK/EUR exchange rate.
4. The swap house raises CZK funds needed in the swap in the domestic market. Greater demand for CZK borrowing may push CZK interest rates up.

**(D)**

CZK interest

EUR interest

**(C)**

EUR principal

CZK principal

CZK

interest

EUR

interest

Swap

house

CZK

principal

**(B)**

EUR

principal

Foreign

market

Czech

government

Domestic

market

EUR principal

**(A)**

CZK taxes

EUR interest

**4 EQUITY SWAP**

**4.1 Description**

equity swap is a derivative contract that commits two counterparties to exchange, over an agreed period of time and with a given frequency, two streams of payments:

* two-way payment linked to percentage changes in an agreed upon stock market index
* one-way payment linked to an agreed upon short-term interest rate

Investor

Bank

LIBOR ± spread

Stock index payments

both income streams are applied to an agreed upon notional principal amount (its purpose is only for calculating the size of the payments exchanged)

only one payment between counterparties is actually made based on the netting procedure

an equity swap is an over-the-counter instrument whose specifications are negotiated and tailor-made by the swap counterparties

**4.2 Synthetic equity investment**

an equity swap can be used to transform money market investment into synthetic equity investment

advantages:

* circumvention of regulatory restrictions on direct investment on the stock market
* purchase of a diversified portfolio of shares in a single transaction that saves transaction costs associated with direct trading in the stock market
* application of an index-tracking investment strategy that avoids rebalancing the managed portfolio in conformity with changes in the benchmark portfolio

**i) Equity swap with fixed notional amount**

Investor

Bank

Money

market

LIBOR

LIBOR ± spread

index payments

repayment at maturity

initial investment

the investor allocates the funds on the money market, but his/her net income (positive or negative ) is derived from the movement of an underlying stock index

|  |
| --- |
| EXAMPLE |
| Complete the table, which summarizes the cash flow of an equity swap with a fixed notional amount of 100 mil USD, semi-annual payment frequency and maturity of two years, given the following development of a six-month LIBOR and an underlying stock index.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | **Month** | **Days in period** | **Stock index value** | **Stock index change (%)** | **LIBOR** | **Index payment** | **Interest payment** | | 0 |  | 2149.40 |  | 13.00 |  |  | | 6 | 182 | 2499.50 | 16.29 | 11.94 | 16,290,000 | 6,482192 | | 12 | 183 | 2420.20 | -3.17 | 10.63 | -3,170,000 | 5,986,356 | | 18 | 183 | 2707.60 | 11.88 | 10.06 | 11,880,000 | 5,329,562 | | 24 | 183 | 2778.80 | 2.63 | 10.00 | 2,630,000 | 5,043,781 | | Sum |  |  |  |  | 27,630,000 | 22,841,890 |   The equity swap earned a profit |

**ii) Equity swap with variable notional amount**

this instrument can be used for the simulation of stock market investment in which profits and losses are capitalised

* equity-linked revenues are reinvested in the money market and equity-linked losses are funded by disinvestment in money market
* a notional principal of equity swap is either increased by an amount equal to the equity-linked revenue or reduced by an amount equal to the equity-linked loss

Investor

Bank

Money

market

LIBOR

LIBOR ± spread

index payments

repayment at maturity

initial investment

reinvestment

disinvestment

increase/decrease

of notional principal

|  |
| --- |
| EXAMPLE |
| Complete the table, which summarizes the cash flow of the equity swap from the previous example with the exception that its notional amount varies in line with the fluctuations of the underlying stock index.   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | **Month** | **Days in period** | **Stock index value** | **Stock index change (%)** | **LIBOR** | **Notional principal** | **Index payment** | **Interest payment** | | 0 |  | 2149.40 |  | 13.00 | 100,000,000 |  |  | | 6 | 182 | 2499.50 | 16.29 | 11.94 | 116,290,000 | 16,290,000 | 6,482,192 | | 12 | 183 | 2420.20 | -3.17 | 10.63 | 112,603,607 | -3,686,393 | 6,961,534 | | 18 | 183 | 2707.60 | 11.88 | 10.06 | 125,980,916 | 13,377,309 | 6,001,279 | | 24 | 183 | 2778.80 | 2.63 | 10.00 | 129,294,214 | 3,313,298 | 6,354,201 | | Sum |  |  |  |  |  | 29,294,214 | 25,799,205 |   The equity swap earned a profit |

**iii) Cross-currency equity swap**

in a cross-currency equity swap, the cash flow streams (based on the stock index changes and the money market rates) are denominated in two different currencies

if the purpose of the swap is to simulate an investment into a foreign stock index, then there is an additional stream of payments that takes account of exchange rate changes in order to preserve the equality between the swap’s notional amount (denominated in foreign currency) and the investment on the domestic money market

an American investor simulates an investment into a euro denominated stock index

Investor

Bank

Money

market

USD LIBOR

USD LIBOR

EUR

index payments

USD repayment at maturity

USD initial investment

reinvestment

disinvestment

USD/EUR compensation

* the euro appreciates against the dollar ⇒ the dollar value of the notional principal amount of the swap, which is denominated in EUR, becomes larger ⇒ the investor’s payments on the USD LIBOR leg of the swap rise above proceeds from the dollar investment on the US money market ⇒ the bank is obligated to compensate the investor by making a payment equal to the difference between the two dollar payments ⇒ investor reinvests the received compensation on the US money market ⇒ the dollar investment on the US money market and the dollar equivalent of the swap’s notional amount are equal again
* the euro depreciates against the dollar ⇒ the dollar value of the notional principal amount of the swap, which is denominated in EUR, becomes smaller ⇒ the investor’s payments on the USD LIBOR leg of the swap fall below proceeds from the dollar investment on the US money market ⇒ the investor is obliged to compensate the bank by making a payment equal to the difference between the two dollar payments ⇒ investor finances this compensation by making a disinvestment on the US money market ⇒ the dollar investment on the US money market and the dollar equivalent of the swap’s notional amount are equal again

**5 SWAPTION**

**5.1 Description**

swaption is an option written on an interest rate coupon swap

* call swaption is the right, not the obligation, to buy an agreed upon forward swap (the swap’s buyer will become the payer of the fix interest)
* put swaption is the right, not the obligation, to sell an agreed upon forward swap (the swap’s seller will become the recipient of the fixed interest)

the option acquirer pays an option premium, which for a swaption is some percentage of the notional principal of the underlying swap

the option holder can exercise the option at a pre-determined strike price, which for a swaption is the interest rate on the fixed leg of the underlying swap.

**5.2 Application**

in the diagram, a company has taken a five-year loan with a floating interest rate, reset every six months, and is concerned about future interest rate increases, so it wants to put a ceiling on its borrowing cost in one year’s time

by buying a one-year call swaption whose underlying asset is a four-year interest rate coupon swap, the company can put a ceiling on its future borrowing rate

Swap dealer

Company

Bank

Interest 11%

6M LIBOR

Loan

6M LIBOR

+100 bp

assuming a swaption premium of 1%, the company can cap its borrowing cost with a fixed ceiling at a level of 13% (the volatile floating rate has no effect on the level of the cap)

13% = 11% (fixed leg of the swap) + 1% (spread in the bank loan) + 1% (swaption premium)

the company can leave the swaption unexercised if it believes that the variable interest rate on the received loan will stay below 13%

the company decides to absorb any increase in the borrowing cost up to 13 %

**6 CREDIT DERIVATIVES**

**6.1 Introduction**

credit derivatives are negotiable financial instruments or techniques that are designed to separate credit risk from underlying (reference) assets and transfer credit exposure between two or more parties

the issuer of the reference asset is not a party to the corresponding credit derivative contract

payoff of credit derivatives is influenced by the occurrence of various types of credit events

* default: inability of the bond’s issuer to repay the outstanding debt
* downgrade: recognized rating agency lowers a credit rating based on an evaluation of the debtor’s earning power to repay the debt
* credit spread widening: the market reaction to perceived credit deterioration by an increased spread over a reference rate
* repudiation: unilateral refusal of the issuer to honour financial obligations
* debt moratorium: postponement of debt repayment by the issuer
* acceleration: enforced repayment of all outstanding debt

general benefits of credit derivatives

* ability to isolate credit risk exposure from reference assets and manage it independently
* savings in transaction costs because using credit derivatives is a cheaper alternative to buying or selling reference assets
* buying credit protection instead of selling deteriorated assets helps preserve business relationships

**6.2 Credit default swap (CDS)**

**i) Description**

CDS buyer

CDS seller

Fee (spread)

Contingent compensation

(premium)

the CDS seller promises to compensate the CDS buyer in the event of a default of a reference asset in exchange for receiving a series of regular payments called fee or spread

* CDS buyer = credit protection buyer = credit risk seller
* CDS seller = credit protection seller = credit risk buyer

the fee (spread) is expressed as a percentage of the notional amount and is paid either over the length of the contract (in case of no default) or until default occurs

compensation for the default can take place in two forms

* cash settlement: the CDS seller pays the CDS buyer the difference between the nominal value (*M*) and the recovery price (*R*) of the reference asset

⇒ investor’s wealth = bond (*R*) + compensation (*M* – *R*) = *M*

* physical settlement: the CDS buyer delivers the asset to the CDS seller for payment of the nominal value of the reference asset (*M*)

⇒ investor’s wealth = bond’s nominal value (*M*) = *M*

analogy with insurance

* CDS buyer pays a fee in return for compensation if an adverse event occurs
* CDS buyer need not suffer a loss from the default because, unlike insurance, he/she need not be the owner of the damaged asset
* CDS seller is not required to maintain reserves to cover protection sold, unlike an insurance company that must create reserves for contingent payments

**ii) Applications**

* hedging credit exposure

Swap dealer

CDS buyer

Reference bond

Compensation

Fee

Total return

investor owns the reference asset and wants to be protected against default

in case of default, the CDS holder receives cash or physical compensation and terminates fee payments

* speculation on credit events

Speculator

Dealer

Fee

Compensation

speculator purchases a CDS in anticipation that the reference asset is about to default and the contingent payment will greatly exceed the fee paid

speculator purchases a CDS in anticipation that the reference asset is more likely to default, so he/she will be able to sell the CDS at a higher fee than paid

naked CDS is speculative purchase of a CDS without holding the reference asset

risks for financial stability (calls for their strict regulation)

* gross amount of CDSes can far exceed the real amount of underlying bonds
* CDS sellers are not required to maintain reserves to cover the protection sold

**iii) Probability model of CDS pricing**

probability model postulates that the price of a CDS is equal to the present value of its expected value, which is the sum of all possible outcomes multiplied by their probabilities with which these outcomes may happen

the CDS price should be zero for a newly issued CDS on efficient markets so the spread is a variable which makes the CDS price equal to zero

|  |
| --- |
| EXAMPLE |
| A reference bond has three years to maturity. It can only default with a probability π at the end of each year, leaving the recovery value of *R*. The CDS owner pays a fee at the end of the year, which is equal to a given percentage *s* of the reference asset’s nominal value *M*. The symbol denotes the discount factor for the period *t*.  All possibilities that might happen during the life of the reference asset are summarized in this branching diagram:  The price for the CDS buyer calculated as a discounted expected value |

**iv) No arbitrage model of CDS pricing**

arbitrage model is based on the argument that a portfolio, composed of a risky par bond and a CDS contract that insures against the bond’s default, is similar to a risk-free par bond

… coupon rate of risky par bond

… CDS spread

… yield of risk-free bond

remarks

* the annuity formula is applied for pricing coupon bonds in which the coupon rate of the risky par bond is reduced by a regularly paid CDS spread
* the yield of the risk-free bond can be used as the discount rate because the portfolio simulates the cash flow of a risk-free bond
* the price of the portfolio is equal to the nominal value of the bond because the portfolio simulates the cash flow of a par bond

equality of the coupon rate and discount rate can be rearranged as

the assumption of a non-existing arbitrage opportunity thus leads to the conclusion that the CDS spread should be set as a difference between the yields of risky and risk-free par bonds.

**6.3 Total rate of return swap (TROR)**

**i) Description**

TROR buyer

TROR seller

Floating rate

Total return

TROR is an agreement to exchange the total return on a reference asset (bond, loan, equity index, etc.) for a floating rate plus a spread

a total return includes both the asset’s income and any appreciation (paid by the TROR seller) or depreciation (paid by the TROR buyer) over the life of the swap, so the total return can be either positive or negative

if a TROR terminates due to default, the buyer can take delivery of the defaulted asset for the initial price (physical settlement) or pay the difference between the initial price and the recovery value (cash settlement)

analogy with leasing

* the TROR buyer gets all of the benefits of and responsibility for damages to the reference asset without being its legal owner
* at maturity of the swap, the TROR buyer can choose to purchase the asset at the prevailing market price

**ii) Applications**

* hedging credit exposure

Dealer

Hedger

Bond

Floating rate

Total return

Total return

by selling the TROR, the hedger, who is the owner of a reference bond, transfers the total return to the dealer in exchange for receiving payments based on a given floating interest rate

in case of a negative return, the hedger is reimbursed for the loss in bond value, in addition to the floating payment

* increasing credit exposure

Speculator

Dealer

Money

market

Bond

Floating rate

Cash

LIBOR

Investment

Total return

Total return

the speculator becomes the recipient of the bond’s total return without the need to own the bond

in case of a negative total return, the speculator compensates the dealer for any depreciation of the bond; alternatively the speculator transfers the collateral to the dealer as a security from which any depreciation of the bond is covered

the TROR exposes the speculator both to the credit risk of the bond (changing credit spreads) and the market risk of the bond (changing market yields), so a credit gain may be offset by a market loss and vice versa

the dealer need not initially own the bond before the TROR is transacted, he/she usually takes a loan to purchase the bond; borrowing costs are reflected in the required floating rate

from the dealer’s perspective the resulting cash flow nets out to a fixed spread over LIBOR

|  |
| --- |
| EXAMPLE |
| A company issued a five-year par bond with a credit spread of 300 basis points over the five-year Treasury whose current yield is 6.5 %. So the bond pays an annual coupon of 9.5%. An investor believes that this spread will decline over the next year. Therefore, he/she concludes a one-year TROR in which he/she receives the total return of the bond (the coupon plus any price appreciation or depreciation) and pays a rate equal to the one-year Treasury, currently 5.7 %, plus 30 basis points. The notional amount of the TROR is 20 million USD.  Assume that after one year the credit spread of the company’s bond narrowed to 270 basis points over the four-year Treasury while the Treasury yield curve remained flat.  The TROR’s cash flow:  coupon payment USD,  interest expenses USD,  USD.  The investor earned the profit  profit USD.  Assume alternatively that the investor was right in anticipating the falling credit spread but the market experienced a general rise in yields. As a result, the yield of a four-year Treasury increased to 7.5%. In this case the bond suffered a capital loss  USD,  and the investor’s profit declined to  profit USD. |

**iii) Leverage**

a TROR enables its buyer to access the total return of the underlying bond at a much lower cost in comparison to the outright purchase of the bond

the degree of leverage is measured by the leverage factor, which is the ratio between the size of the risky exposure and the amount of invested capital

leverage can significantly boost the return on the invested capital but also cause substantial losses in case of an unfavourable development

|  |
| --- |
| EXAMPLE |
| Compare yields to maturity of the following two investment strategies:   * Purchasing and holding a three-year par bond with a coupon of 6% and a nominal value of 100 USD. * Arranging a three-year TROR with the same reference bond and a fee of three percentage points above the three-year Treasury. The TROR buyer has to put up collateral of a three-year Treasury in an amount of 10% of the face value of the bond (leverage factor is 10).   The strategy of purchasing the bond.  The bond in question is the par bond, so its price is equal to the bond’s face value. The yield to maturity can be found by solving the equation:  We have YTM = 6% because the discount rate is equal to the coupon rate for par bonds.  If the bond suffers a capital loss of 10% of face value, the YTM would be determined by the equation  which gives a smaller but still a positive yield of 2.8%.  The strategy of purchasing the TROR.  The investor receives a coupon of 6%, pays a fee of 3% above Treasury and retains the yield of the collateral, which is the Treasury’s yield. So in net terms the investor gets 3% of the bond’s face value (+ 6% – (T+3%) + T = + 3%), although his/her actual investment is only 10 USD. The equation for the yield to maturity is thus  The leverage results in a substantial higher YTM of 30%.  In the case of a capital loss of 10%, the investor loses all collateral. The YTM equation becomes  giving the solution YTM = – 5%. |

arrangement of the deal with the assistance of a Special Purpose Vehicle

* an SPV is a subsidiary of an investment bank set up for the purpose of issuing securities whose returns are tied to an underlying pool of bonds
* an SPV engineers a highly leveraged derivate deal designed as a conventional purchase of debt securities

Investment

bank

SPV

Treasury

market

Investor

Total return

10 million

Treasury

yield

SPV securities

LIBOR + spread

10 million

Bond

market

100 million

Total return

Money

market

100 million

LIBOR

**(A)**

**(B)**

**(C)**

**(D)**

**(D)**

1. An investor purchases SPV securities in a nominal value of 10 million USD whose return is based on the pool of reference bonds with a face value of 100 million USD (leverage factor is 10).
2. An SPV uses proceeds from the sale of SPV bonds to buy Treasury notes that serve as collateral against a decline in value of the reference pool of bonds. Yield on Treasury notes is transferred to the investor minus (LIBOR + spread + fee).
3. An SPV collets the total return on the reference pool from the investment bank, which is passed to the investor as a yield on SPV securities. The SPV earns a spread over the LIBOR without being exposed to the credit risk of the reference pool of bonds.
4. An investment bank borrows 10 million USD on the money market.
5. The investment bank uses the borrowed amount to purchase the pool of the underlying bonds whose total return is passed on to the SPV. The bank earns a spread over the LIBOR and is protected against credit risk stemming from the purchased pool of bonds.

**6.4 Credit options**

credit options give the investor the right, not the obligation, to buy (a credit call) or sell (a credit put) a specific credit risk at a given exercise price

the option buyer pays an option premium for acquiring the option

two types of credit options:

* credit level options
* credit spread options

**i) Credit level options**

credit level options design the pay-off as the difference between the market price and the specified strike price of a reference bond

credit risk is incorporated into these options through the size of the strike price which is specified as a bond’s price that corresponds to a given credit spread over a referenced bond

payoff formulas and diagrams of credit level options look the same as those of standard options

credit level long call credit level long put

*X*

*ST*

*-P*

*X*

*ST*

*-C*

… option payoff

… call option premium

… put option premium

… bond’s price at option’s maturity

… strike price corresponding to a specified spread over a risk-free bond

... yield of the reference bond

... strike spread over a risk-free bond

|  |
| --- |
| EXAMPLE |
| A portfolio manager purchased a five-year zero-coupon bond at a nominal value of 1,000 USD and at a credit spread of 215 basis points over a comparable Treasury, which is now traded at a yield of 6.25%. To be protected against a widening credit spread, the manager also purchases a one-year European credit level put option whose exercise price is established at a credit spread of 225 basis points.  The strike price of the option is thus set equal to  If in one year’s time the bond is traded below this value, the manager can exercise the option and receive the difference between 721.57 USD and the current market value of the bond. |

**ii) Credit spread options**

credit spread options design the pay-off as the difference between two spreads over the yield of a given risk-free bond

one of these spreads is the market spread at the time of exercising the option and the other one is a predetermined exercise spread

in pay-off formulas of credit spread options, the difference of spreads is multiplied by a modified duration (eventually plus convexity) of the reference bond, which is the application of a well-known relationship, approximating the change in the bond’s price in response to a change in its yield

the inverse relationship between the price and the yield of the bond is the cause of the flipped feature of payoff profiles between credit level and credit spread options

*credit spread long call* *credit spread long put*

*x*

*sT*

*-P*

*x*

*sT*

*-C*

... market spread at option’s maturity

... strike spread

... modified duration of the reference bond

... face value of the reference bond

|  |
| --- |
| EXAMPLE |
| A portfolio manager believes that the current credit spread, which is currently 170 basis points over the Treasury, will increase over the course of the next year. Therefore, he/she decides to buy a credit spread put with the tenor of one year and a strike spread of 170 basis points. The option premium is 120 basis points, the bond’s modified duration is 6.65% and the bond’s nominal value is 20 million USD.  What would the payoff be from the option if the credit spread widens to 250 basis points at maturity of the option?  The payoff from the credit spread long put is |

**iii) Links between credit level and credit spread options**

payoffs of both types of credit options are linked through the approximation formula for a change in the bond’s price in response to a change in its yield

assumptions

* the yield of the risk-free bond remains unchanged at a value of
* a change in the reference bond’s yield is expressed as a change in the spread over the yield of the risk-free bond
* a reference bond is a par bond
* denote the prices of the reference bond that correspond to the yields respectively
* the initial bond price is equal to the strike price
* denote the strike spread and the market spread at the option’s maturity

similarity of payoffs results from the following set of adjustments

**6.5 Credit forwards**

credit forwards have features of standard forward contracts that are adapted to managing credit risk (no upfront payment, no protection against unlimited loss)

credit forwards can be constructed, like credit options, either on a level basis (the payoff depends on the difference between two prices) or on a spread basis (the payoff depends on the difference between two spreads)

*long credit spread forward* *short credit spread forward*

... market spread at option’s maturity

... strike spread

... modified duration of the reference bond

... face value of the reference bond

**6.6 Credit linked notes**

credit linked notes (CLNs) are hybrid instruments that combine elements of a debt instrument and credit derivatives (credit options, credit forward or both)

bonds with such embedded credit components are often called structured bonds

examples

* a CLN with an embedded short credit level put option

payoff characteristics

option premium provides an incremental income to the note’s coupon

if the credit rating on the reference note declines below the grade BBB the note buyer receives a lower total income

Note’s coupon

CLN payoff

Short credit level put

* a CLN with an embedded credit spread put, credit spread call and credit spread forward

payoff characteristics

if the spread widens, the CLN generates income in excess of the bond’s principal value, capped at a spread of 300 bp

if the spread narrows, the CLN generates income below the bond’s principal value, stopped at a spread of 100 bp

between the spread range of 100 and 300 pb the income is allowed to fluctuate in response to the spread change

Note’s face value

CLN payoff

Long credit spread call

Short credit spread put

Short credit spread forward

**6.7 Collateralized debt obligations**

**i) Subordination structure**

collateral debt obligations (CDOs) are the products of a securitization technique that is based on the use of the subordination principle in managing credit risk

subordination structure consists of classes of securities, called tranches, organized in a hierarchical sequence, called a waterfall, that absorb the losses from the defaults of the underlying bonds

proceeding from the safest to the riskiest one, we use the names senior tranche, mezzanine tranche, junior tranche and equity tranche

each tranche is allotted a percentage representation in the total volume of CDOs issued

Bond 1

Bond N

Collector (structurer)

Senior tranche (75%)

Equity tranche (5%)

Mezzanine tranche (10%)

Junior tranche (10%)

pecking order in absorbing losses

* when bonds start defaulting, it is the equity tranche that absorbs the losses first; it is completely wiped out if default losses exceed 5% of the principal amount of the underlying pool of bonds
* the junior class is hit when the default rate rises above 5% and is exhausted when defaults reach 10%
* the mezzanine class is hit when the default rate rises above 15% and is used up if defaults reach 25%
* the senior class is residual and is hit when the default rate rises above 25%

basic properties of the subordinate structure of CDO tranches.

* the lower the tranche in the waterfall, the more it is exposed to credit risk; a higher risk will then be reflected in a higher promised yield
* the probability that the tranche will be hit by losses largely depends on the size of all CDO classes lower on the hierarchy; with the appropriate design of these sizes, the securities in senior tranches can become an almost risk-free asset
* the assumption about the strength of interdependency among defaults of individual bonds in the underlying pool is critically important; the lower the correlation, the stronger the diversification effect, and therefore the lower the risk of losses for CDO tranche holders
* in a time of crisis, holders of senior bonds may be affected by downgrades of this class even though the senior class does not absorb any default losses; the reason for downgrade could be the financial distress of the collector of the underlying assets, the drying up of liquidity on financial markets, the general loss of confidence in securitization techniques, etc.

**ii) Risk-return properties of CDO tranches**

* a collector assembled a pool of ten risky bonds, each of them having a 10% probability of default and zero recovery value; with a 90% probability, the bond will not default and will pay the entire face value of 100€ at maturity
* default characteristics of an individual bond

|  |  |  |
| --- | --- | --- |
| Number of defaults | 0 | 1 |
| Probability of default (%) | 90 | 10 |
| Loss (EUR) | 0 | 100 |

* risk-return characteristics of an individual bond

|  |  |
| --- | --- |
| Expected loss (EUR) | 10 |
| Standard deviation (EUR) | 30 |

* a subordination structure consists of ten CDOs, of which two form the junior tranche, another two belong to the mezzanine tranche and the remaining six are included in the senior tranche; the nominal value of one CDO is 100 EUR , the nominal values of the left side and the right side of the balance sheet are equal.

Bond 1

Bond 10

Collector (structurer)

Senior tranche – 6 CDOs

Mezzanine tranche – 2 CDOs

Junior tranche – 2 CDOs

* default characteristics of the CDO structure

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number of defaults | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Probability of default (%) | 34.87 | 38.74 | 19.37 | 5.74 | 1.12 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Loss per one junior CDO | 0 | 50 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| Loss per one mezzanine CDO | 0 | 0 | 0 | 50 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| Loss per one senior CDO | 0 | 0 | 0 | 0 | 0 | 17 | 33 | 50 | 67 | 83 | 100 |

* risk-return characteristics of the CDO structure

|  |  |  |
| --- | --- | --- |
|  | Expected loss | Standard deviation |
| Junior tranche | 45.76 | 27.23 |
| Mezzanine tranche | 4.15 | 15.94 |
| Senior tranche | 0.03 | 0.77 |

it’s noteworthy that for the senior tranche the expected loss is virtually nil and the associated risk is also negligible

an investment in the mezzanine CDO looks safer than an equally large investment in one risky bond